PLANE TRUSSES

Definitions

A truss is one of the major types of engineering structures which provides a practical and economical solution for many engineering constructions, especially in the design of bridges and buildings that demand large spans.

They consist of straight members i.e. bars, connected at their extremities through joints. Therefore no member is continuous through a joint.

All the members lie on a plane, while the **loads** carried by the truss, **are only concentrated forces** that **act on the joints** and lie on the **same** plane.

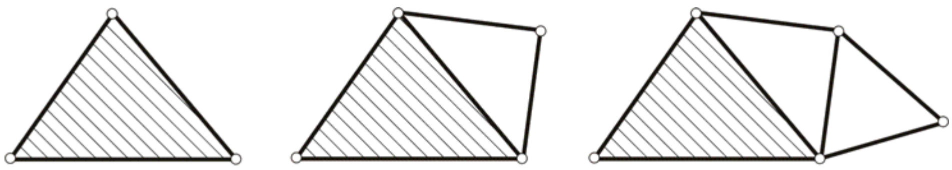
When a concentrated load is to be applied between two joints, or a distributed load is to be supported by the truss – as in the case of a bridge truss – a floor system must be provided, in order to transmit the load to the joints.

Although the members are actually joined together by means of bolted or welded connections, it is assumed that they are pinned together. So the forces acting at each end of a member are **only axial**, without the existence of bending moments or shear forces.

Each member can be treated as a two-force member, in which the two forces are applied at the ends of it. These forces are necessarily equal, opposite and *collinear* for equilibrium.

The entire truss can therefore be considered as a group of pins and two-force members, which obviously are **either in tension or in compression**.

The basic element of a plane truss is the triangle. Three bars jointed by pins at their ends constitute a rigid frame. The structure may be extended by adding each time two additional bars through a joint to form a rigid, i.e. noncollapsible structure.



Structures that are built from a basic triangle in this manner are known as *simple trusses*.

Trusses that are geometrically similar and have the same loads at corresponding joints, will present equal forces to the respective members. This means that the force of a member is not dependant on the size of the truss itself but on the magnitude of the external loads and the geometry of the truss.

When more members are present than those needed to prevent collapse, the truss is statically indeterminate. On the other hand, when fewer members are present, the truss is not rigid, forming a mechanism.

A truss is said to be rigid and statically determinate, when the number of members, m, along with the number of joints, j, satisfy the equation

$$m = 2j - 3 \ .$$

The concept of rigid expresses the stability of the truss, without being a mechanism, while the term statically determinate defines the possibility for the truss to be analyzed and solved through one of the three methods that will be presented hereafter.

The term analysis and solution of a truss, denotes the necessary procedure, to find for all or some of the members:

- The magnitude of the axial force and
- The situation of act for each member, i.e. if it is under tension or compression.

The three methods to solve a truss are:
1. The analytical method of joints
2. The graphical method of Cremona's diagram and
3. The method of sections

The method of joints

This method demands satisfaction of the conditions of equilibrium for the forces acting on the connected pin of each joint. The method therefore deals with the equilibrium of concurrent forces acted on the joint, where only two independent equations are involved:

$$\sum x^{\overset{+}{\rightarrow}} = 0 \qquad \text{and} \qquad \sum y^{\uparrow +} = 0$$

The equation $\left(\sum M\right)^{\circlearrowleft +} = 0$ cannot be used, once the forces are concurrent.

We start the analysis with any joint, where at least one known load exists and not more than two unknown forces are present.

The external reactions are usually determined by applying the three equilibrium equations to the truss **as a whole**, before the force analysis of the truss is begun.

During the equilibrium analysis of a joint, when we introduce the unknown force of a member, the arrow which expresses the sense of its vector is arbitrary. In this way, if the sense of the arrow is away from the pin, this means that the member pulls the joint, i.e. the bar is under tension; otherwise it pushes the joint, i.e. it is under compression.

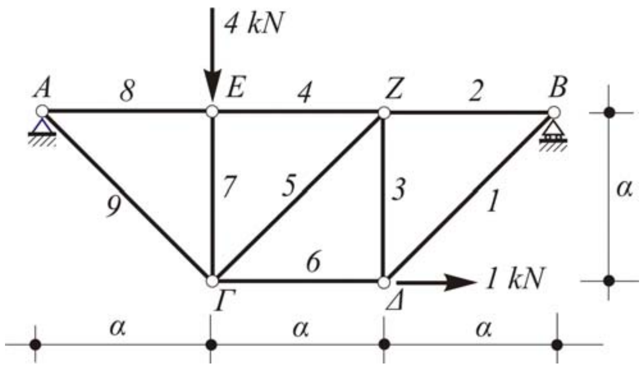
The positive or negative sign that yields from the equation of equilibrium, denotes respectively the correct or wrong sense of our arbitrary choice.

If three forces act on a joint, and the two of them are on the same line, while the third one is vertical or forms any angle with that line, then the third force is always **zero**, while the other two are **equal and opposite**.

The procedure of the method is presented in the example that follows.

Example

Compute the force in each of the nine members of the following truss by the method of joints.



Solution

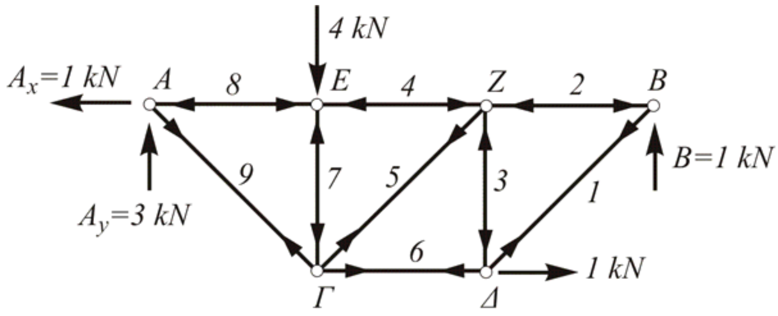
Reactions

$$\left(\sum M\right)_A^{\circlearrowleft +} = 0 \;\Rightarrow\; 4 \cdot a - 1 \cdot a - B \cdot 3a = 0 \;\Rightarrow\; \underline{B = 1\text{ kN}}$$

$$\sum y^{\uparrow +} = 0 \Rightarrow A_y - 4 + 1 = 0 \quad \Rightarrow \quad \underline{A_y = 3\text{ kN}}$$

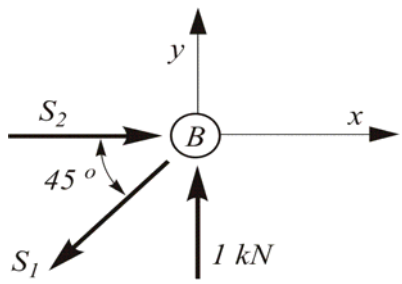
$$\sum x^{\overset{+}{\rightarrow}} = 0 \Rightarrow -A_x + 1 = 0 \quad \Rightarrow \quad \underline{A_x = 1\text{ kN}}.$$

Having calculated the reactions, we draw the free body diagram of the truss and start analyzing the joint equilibrium, where, concurrent are only two unknown forces.



Equilibrium of joint B

We design the joint B as the zero point of a virtual Cartesian coordination system, by drawing **all** the forces that act on it (here, completely known is the reaction B = 1 kN, while the other two are known in direction only), introducing, for instance, S_1 in tension and S_2 in compression.



Starting from equation $\Sigma y^{\uparrow +} = 0$, (in order to avoid S_2), we get:

$$\sum y^{\uparrow +} = 0 \Rightarrow 1 - S_1 \cdot \sin 45^o = 0 \text{ or } S_1 = \frac{1}{0.707} = +1.41\text{ kN},$$

$$\sum x^{\overset{+}{\rightarrow}} = 0 \Rightarrow S_2 - 1.41 \cdot \cos 45^o = 0 \text{ or } S_2 = 1.41 \cdot 0.707 = +1\text{ kN}.$$

The fact that the sign yielded for the forces S_1 and S_2 is positive, means that the senses we selected for these forces are correct.

These **correct** senses are now transferred on the corresponding members of the free body diagram, **beside** the joint, whose equilibrium has already been analyzed.

According to the principle of action – reaction (Newton's third law), we then draw at the other ends of the same members (1 and 2) the opposite senses, which are the real actions on the adjacent joints.

Now we notice that member 1 pulls joint B. Therefore it is under tension of 1.41 kN and also pulls the adjacent joint Δ, by the same force.

At this time, on the table that follows at the end of this solution, we record the result for member 1 as +1.41 kN.

Member 2 on the contrary pushes joint B. Therefore it is under compression of 1 kN and also pushes the adjacent joint Z, by the same force.

This new result is recorded on the table as –1 kN.

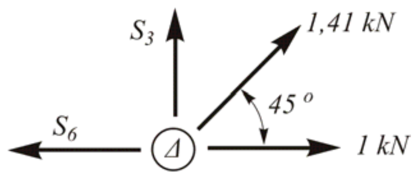
Since on the joint Z concurrent are three unknown forces, we have to move to the joint Δ, following the same procedure.

Equilibrium of joint Δ

Here, introducing both the unknown forces as tensile, we get:

$$\sum x^{\overset{+}{\rightarrow}} = 0 \quad \Rightarrow \quad -S_6 + 1.41 \cdot \cos 45^o + 1 = 0 \quad \Rightarrow \quad \underline{S_6 = +2\ \text{kN}},$$

$$\sum y^{\uparrow +} = 0 \quad \Rightarrow \quad S_3 + 1.41 \cdot \sin 45^o = 0 \quad \Rightarrow \quad \underline{S_3 = -1\ \text{kN}}.$$



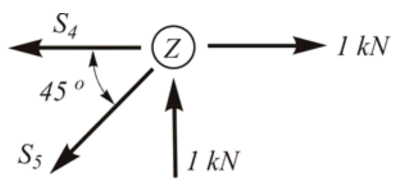
While the positive sign of S_6 means the correct sense of its vector, the **negative** sign of S_3 means that the correct sense of this vector is the **opposite** from what has been selected, in other words the member is under **compression**.

Transferring the correct senses on the corresponding members of the free body diagram beside the joint Δ and following the same procedure as before, we record on the table the results for members 6 and 3 as $+2$ and -1 respectively.

Note: From the above sequence it is clear that if we initially introduce an unknown force as **tensional**, then **the sign that yields** from the equilibrium equation **directly expresses the real situation of the corresponding member**. The opposite occurs if we introduce the force as compressional.

Equilibrium of joint Z

Similarly, from the following figure, we get:

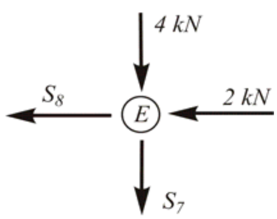


$$\sum y^{\uparrow +} = 0 \quad \Rightarrow \quad 1 - S_5 \cdot \sin 45^o = 0 \quad \Rightarrow \quad \underline{S_5 = +1.41\ \text{kN}},$$

$$\sum x^{\overset{+}{\rightarrow}} = 0 \quad \Rightarrow \quad -S_4 - 1.41 \cdot \cos 45^o = 0 \quad \Rightarrow \quad \underline{S_4 = -2\ \text{kN}}.$$

Equilibrium of joint E

Similarly, from the following figure, we get:

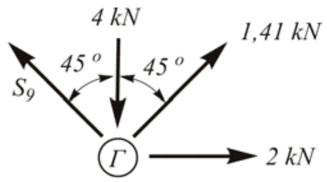


$$\sum x^{\overset{+}{\rightarrow}} = 0 \quad \Rightarrow \quad -S_8 - 2 = 0 \quad \Rightarrow \quad \underline{S_8 = -2\ \text{kN}}$$

$$\sum y^{\uparrow +} = 0 \quad \Rightarrow \quad -S_7 - 4 = 0 \quad \Rightarrow \quad \underline{S_7 = -4\ \text{kN}}$$

Equilibrium of joint Γ

Since we have here only one unknown force, the other equation will be used for checking.

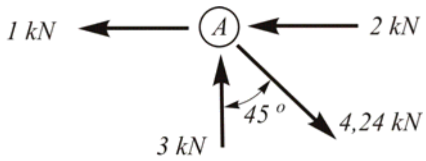


$$\sum x^{\overset{+}{\rightarrow}} = 0 \quad \Rightarrow \quad -S_9 \cdot \sin 45^o + 1.41 \cdot \sin 45^o + 2 = 0$$

$\Rightarrow$ $\underline{S_9}$ = (1 + 2)/0.707 $\underline{= \ +4.24 \text{ kN}}$

Checking: $\sum y^{\uparrow +} = 4.24 \cdot \cos 45^o - 4 + 1.41 \cdot \cos 45^o = 3 - 4 + 1 = 0!$

Equilibrium of joint A (Checking)



$$\sum x^{\overset{+}{\rightarrow}} = -1 + 4.24 \cdot \sin 45^o - 2 = -1 + 3 - 2 = 0!$$

$$\sum y^{\uparrow +} = 3 - 4.24 \cdot \cos 45^o = 3 - 3 = 0!$$

Table denoting the force of each member

Member	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Force (kN)	+1.41	– 1	– 1	– 2	+1.41	+ 2	– 4	– 2	+4.24

Having located the correct senses for all forces on the free body diagram, we note that for each member, the real axial force is the opposite from what the member initially tends to show. For example, member 2, while tends to show that it is under tension, in reality it is under compression, because it pushes both joints at its ends.

The checking, that is realized at the end of the procedure, is not necessarily a part of the solution of the truss. However, when the check is done and holds, it shows that both the reactions and the member forces have been correctly calculated.

The designer obtains therefore the necessary confidence to follow the next stage of construction design, which is the calculation of the necessary cross section for each member, taking into account its material properties etc.

The equations that have been used for checking, are substantially redundant and come as a result from the fact that we have already used the equations of equilibrium for the truss as a whole to calculate the reactions.

Indeed, in a truss with j joints, and m = 2j – 3 member forces to be calculated, if we add the 3 unknown reactions that appear to a statically determinate girder, we totally obtain 2j – 3 + 3 = **2j** unknown forces, that can normally be calculated through the **2j** equations, yielding from the equilibrium of each joint.

The reason that we first calculate the reactions, is mainly to start the procedure of joint equilibrium from one of the **reaction joints**, where, there are usually only 2 unknown member-forces exerted.

The Cremona's graphical method

This method deals mainly with the graphical representation of equilibrium for each joint. The basic advantage that makes the method attractive, is its ability to unify all the force polygons, resulting from graphical equilibrium of each joint, **into one only force polygon**, known as *Cremona's diagram*.

Although graphical, this method leads to a quick determination of the member forces and is useful specifically in the cases where the external loads and/or the truss members form random angles.

Consider the case of graphical analyzing the equilibrium of a point, acted upon 3 forces, one of which is completely known while the other 2 are known in direction only (for example, a lamp hanged by two wires).

All we have to do is:

a) Draw the vector of the completely known force, in the proper direction, scale, magnitude and sense.

b) From **one end** of the vector, draw a line parallel to the direction of **one** of the 2 forces, while from the **other end** draw a second line parallel to the **other** direction. The vector and the point of section of the two lines define a triangle.

c) Now, following the path of the vector by laying out the 2 unknown forces tip to tail, thus closing the force triangle, we find both the **magnitudes** and the **senses** of the other 2 forces.

Of course the completely known force can be considered as the resultant of other known forces, through a force polygon.

From this procedure we realize that the basic characteristic which appears to be common in the method of joints and Cremona's diagram lies in the main strategic. For analyzing the equilibrium of a joint, in the first method available were **2 equations** only, whereas in the second, the **two ends** of the known-force-vector only.

Keeping in mind this similarity for the new method, we can also start and continue with the equilibrium of a joint, where at least one known load exists, while not more than two unknown forces are present.

Compared to the analytical method of joints, the graphical method of Cremona's diagram is less precise. However, the 'loss of precision' is unimportant and theoretical. Nevertheless, the speed and the elegance of the method are the main characteristics that make it popular and attractive by many designers.

In organizing the method, specifically in naming the vectors of the diagram that express the member forces, significant was the contribution of **Bow**. This is the reason that the whole procedure is also known as method of Bow – Cremona.

In the example that follows, the different stages of the method give the impression of a sophisticated work. However, having obtained some experience, these stages are followed mechanically and the graphical solution is realized quickly and safely.

Example

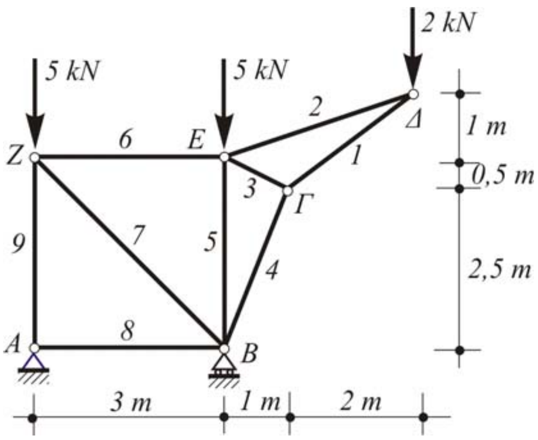
Determine graphically the force in each of the nine members of the following truss (next page) by the method of Cremona's diagram. Check the results.

Solution

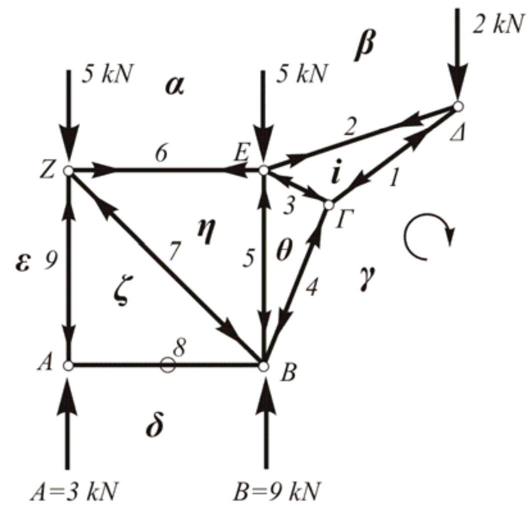
Reactions

$$(\Sigma M)A^{\circlearrowleft +} = 0 \quad \Rightarrow \quad 5\cdot 3 + 2\cdot 6 - B\cdot 3 \;=\; 0 \qquad \Rightarrow \qquad \underline{B \;=\; 9\,\text{kN}\,,}$$

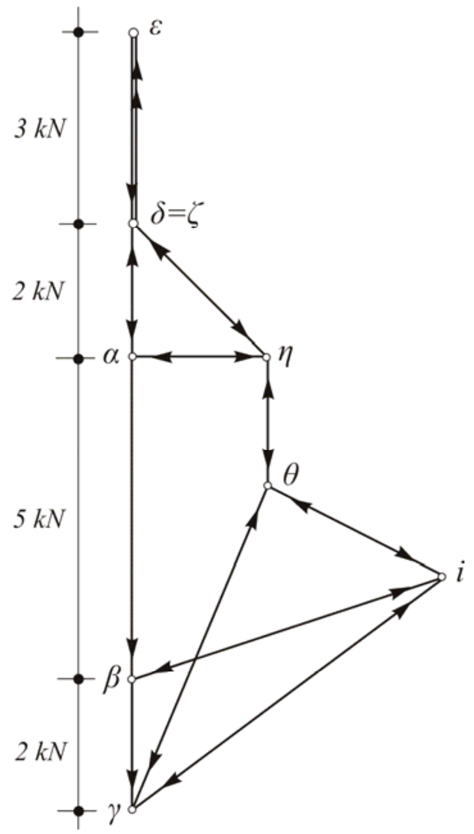
$$\sum y^{\uparrow +} \;=\; 0 \quad \Rightarrow \quad A - 5 - 5 + 9 - 2 \;=\; 0 \Rightarrow \qquad \underline{A \;=\; 3\,\text{kN}\,.}$$



Having calculated the reactions, we draw the free body diagram (figure *α*) and follow the next steps:



(α) Free body diagram



(β) Cremona's diagram

a) We define a **clockwise sequence** of forces around a joint. This means that if we start drawing the force triangle for equilibrium of joint Δ, from, say, the known force of 2 kN, the next force considered will be that of member 1 and not of 2.

b) Covering the whole area of the free body diagram, we name, say with Greek letters α, β, γ… both the triangles formed by the members and by the external loads, so that each member or load separates two areas.

c) Then we start the equilibrium of a joint, say Δ, where only two unknown forces are concurrent, while the rest are known.

d) Keeping the clockwise sequence, we always consider **first the known loads** to **end up with the two unknown forces**.

Joint Δ

Defining the scale, we draw the known force of 2 kN by the **named vector** $\overrightarrow{\beta\gamma}$, because, **rotating clockwise** with respect to **joint Δ**, before crossing the load of 2 kN, we **first** step on the area ***β*** and **then** on the area ***γ***.

Going on clockwise, we meet the **member 1.** The force that this member exerts to the joint Δ is ***γι***, due to the areas *γ* and *i* that it separates (clockwise with respect to Δ).

From point *γ* on the Cremona's diagram we draw a parallel to member 1. On this parallel we **expect** the point *i*.

Continuing clockwise, we meet the **member 2**, which similarly exerts to the joint Δ the force *ιβ*.

Since the point *i* is not yet known, if from point *β* we draw the parallel of member 2, it crosses the previous parallel to member 1 at the point *i*.

Having defined this point of section, the equilibrium of joint Δ is now expressed through the force triangle *βγiβ.* To close the polygon we follow the path yielding from the area letters *γiβ* that correspond to a **clockwise rotation above the last two** unknown forces, **putting the arrow of sense at the end** of each vector.

These arrows express the correct senses of the unknown forces. Next we transfer the correct arrows to the corresponding **members** 1 and 2 on the free body diagram (figure *α*), **beside** the joint Δ, whose equilibrium has been considered.

On the same members, we put next the **opposite** arrows at the other ends, i.e. close to the joints *Γ* and *E*.

Now we notice that member 1, for instance, pushes joint Δ, with a force which is defined from Cremona's diagram, if we measure – according to the scale – the length *γi* (or *iγ*).

We find therefore *γi* = 6 units, and, on the table that follows, we denote the force of member 1 by the number –6.

Table denoting the force of each member

Member	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Force (kN)	– 6	+ 5.1	– 3.1	– 5.4	– 2	+ 2	– 2.8	0	– 3

Similarly we find that member 2 pulls the joint Δ by a force *iβ* = 5.1 kN. We record the force as + 5.1 and move on to the next joint Γ, once on the joint E there are 3 unknown forces.

Joint Γ

In order to end up with the two unknown forces of members 3 and 4 (figure *α*), we start our path from the known force of member 1, which is now *iγ*.

We put therefore (figure *β*), the arrow of sense on the **empty** final part of length *iγ* (1^{st} check), which arrow is the same with that placed before on figure *α,* as opposite at the other end of member 1.

Going on clockwise we meet member 4, which exerts force *γθ* to joint Γ. From *γ* we draw a parallel to member 4, where we **expect** the point *θ*.

We next meet member 3 (exerting to the joint the force *θi*). From *i* we draw a parallel to 3, thus defining the point *θ*.

On the force triangle *iγθi*, which now expresses the equilibrium of joint Γ, we follow the path over the two unknown forces, i.e. *γθi*, putting at the end of the vectors the arrows of sense to this path.

These arrows are then transferred to the corresponding members (figure α), after which we put the opposite arrows at the other ends.

We find out that members 4 and 3 are under compression. Measuring the lengths $\gamma\theta$ and θi, we record on the table the forces –5.4 and –3.1 respectively.

Joint E

We start the force diagram from the known load $\overrightarrow{a\beta}$ = 5 kN, so that the two unknown forces 5 and 6 are the **last** ones. From the point β we already have, we can easily define point α, by taking vertically upwards the length $\beta\alpha$ = 5 units.

Going on clockwise, we meet the already known force βi of member 2. Then, moving to Cremona's diagram we find at the end of length βi the **empty** space to put the arrow of sense, which is checked with the one that we put before in figure *(α)*.

In the same way we check the sense of the known force $i\theta$ of member 3.

According to what presented before, from point θ (figure β), we draw the parallel to member 5, which crosses the parallel to 6 from α, at point η.

On the Cremona's diagram, we cover the path $\theta\eta\alpha$, laying the arrows at the end of each direction, which in turn are transferred to the corresponding members close to the joint, after which we put the opposite arrows to the other ends.

On figure β, measuring the new lengths $\theta\eta$ and $\eta\alpha$, and checking the push or pull of members 5 and 6 we record them on the table as –2 and +2 respectively.

Joint Z

In a similar way we draw on figure *(β)* the force $\overrightarrow{\varepsilon a}$ = 5 kN defining the point ε, and check the force $\alpha\eta$ of member 6.

From η we draw a parallel to 7, crossing the parallel to 9 from ε at the point ζ.

The length $\eta\zeta$ defines the force –2.8 kN of member 7, while the length $\zeta\varepsilon$, which is **double** (drawn here with a double line), defines the force –3 kN of member 9.

Joint A

Having moved on the joint A we expect to find a zero-force for the member 8, according to the last paragraph of the method of joints.

Indeed, having put on the Cremona's diagram the known reaction A = $\overrightarrow{\delta\varepsilon}$ = 3 kN, we define the point δ, belonging to the downwards vertical line from ε at a distance of 3 units. However, since the force of member 9 has already been determined to 3 kN, it follows that δ **coincides** with ζ. The force therefore for member 8 (expressed by the length $\zeta\delta$) is zero.

Joint B

The procedure of graphical equilibriums will be completed at the joint B, by checking the already known reaction B, i.e. its magnitude, direction and sense.

Indeed, starting from the zero force of member 8 and checking the forces for members 7, 5 and 4 respectively with the vectors $\overrightarrow{\zeta\eta}$, $\overrightarrow{\eta\theta}$ and $\overrightarrow{\theta\gamma}$, we find out that the magnitude $\gamma\delta$ of reaction B is in fact 9 kN (= 9 units) with a vertical direction and an upwards sense.

Note: The calculation of reactions, especially for this truss could be avoided, once, for the start of graphical equilibrium, there is a joint (Δ), where there is a known load and only two unknown forces. In such cases the reactions **yield** from the equilibrium conditions at the **support joints**, and normally are used for checking.

The method of sections

A truss is possible to be rigid and statically determinate, i.e. the equation

$$m = 2j - 3$$

may be satisfied. However, the truss itself might present problems in finding the forces of its members. In some cases these problems may be overcome after some effort, but in some other cases not. In particular:

a) It is possible the force only of one member (or more) of the truss to be demanded, and this member(s) might be **far** from the joint, where we can start the conditions of equilibrium.

b) The **start** of applying the equilibrium conditions **may be impossible for any joint** once there are at least 3 unknown concurrent forces to the joints.

c) We start normally the application of equilibrium conditions, but progressively, we reach a point, where the continuation of the procedure is **impossible** because 3 unknown forces are concurrent on any joint.

In all the above cases, and not only, the solution is given by the analytical method of sections, or the **Ritter's** method.

The steps we usually follow to proceed on the determination of one or more member-force(s) of the truss are:

1. Having calculated the **reactions** of the truss, pass a section through **three** members of the truss, one of which is the demanded for calculation. In this way we obtain two separate portions of the truss.
2. Put **tensile** forces at all intersected members, so that the sign which yields after the calculation expresses the **real** axial force of the member.
3. Of the two portions, select the one having the **smaller** number of external loads and draw the free body diagram.
4. In general, write three equilibrium equations, which can be solved for the forces of the three intersected members. It should be noted here that we prefer to use the equation $\left(\sum M\right)_i \,{}^{\circlearrowleft +} = 0$, with respect to the **point of section *i***, between **any two** of the three intersected members, to find the force at the **third** one. A minor difficulty might appear here, to calculate the moment arm of the third member.

Before we pass the section, we make sure that:

1. We do not separate a joint, but a portion of the truss, not less than a triangle.
2. The number of intersected members should not be more than 3, if we want to avoid complicated calculations. However, since the minimum number of members to secure rigidity between two portions is 3, we prefer this number to be **3**.

In the examples that follow, the procedure of the method is presented, to face only the described problem. A further solution of the truss, if demanded, can be normally realized through one of the two preceding methods.

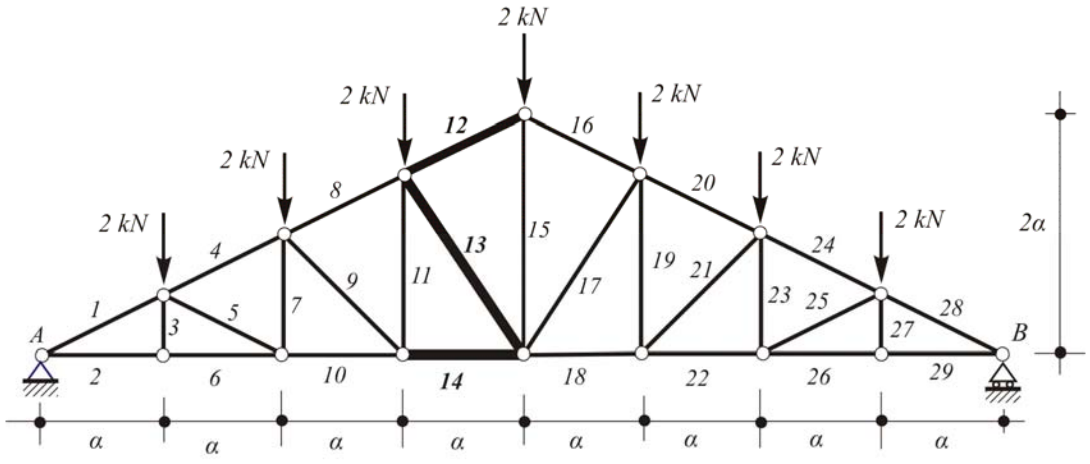
Examples

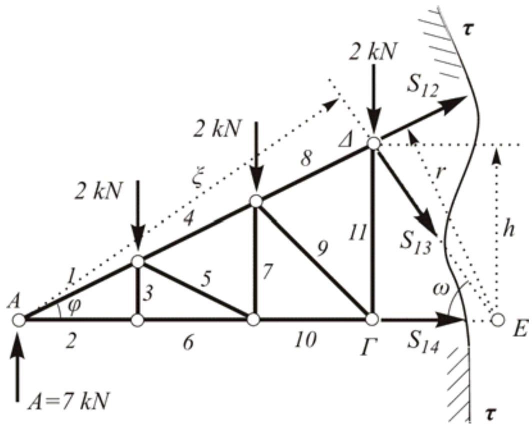
1

Compute the force in each of the three members 12, 13 and 14 of the following truss (next page) by the method of sections.

Solution

After the calculation of reactions, which here, due to symmetry, are **A = B = 7 kN**, we pass a section through members 12, 13 and 14, thus separating the truss into two portions.





Drawing the free body diagram for the left portion, we notice that, before section, the portion ΑΓΔ of the truss was in equilibrium under the action of the known reaction A = 7 kN, the three vertical forces of 2 kN each, and, of course, those corresponding to the intersected members 12, 13 and 14.

The equilibrium of the above portion will continue to exist only if at the place of intersected members we put the real but still unknown (internal for the truss) forces, i.e. S_{12}, S_{13} and S_{14} respectively, which we introduce tensile. If the yielding sign for a force is negative, this will mean that the real force is opposite, i.e. compressive.

If ***h*** is the distance from **Δ** (cross point of members 12 and 13) to the member 14, ***r*** the distance from **E** (cross point of members 13 and 14) to the member 12 and ***ξ*** the distance from **A** (cross point of members 12 and 14) to the member 13, since the portion ΑΓΔ is in equilibrium, the three equilibrium equations will hold, i.e.

$$\sum x^{\overset{+}{\rightarrow}} = 0, \quad \sum y^{\uparrow +} = 0, \quad \text{and} \quad \left(\sum M\right)_i^{\circlearrowleft +} = 0.$$

We prefer instead to use three times the last equation with respect to the points Δ, E and A, since the moment arms h, r and ξ of the respective unknown forces S_{12}, S_{13} and S_{14} can easily be calculated trigonometrically. Hence:

$$\tan\varphi = \frac{1}{2} = \frac{h}{3a} \quad \Rightarrow \quad \text{and} \quad \underline{h = 1{,}5\,a}$$

$$\varphi = 26{,}57^{\circ} \quad \Rightarrow \quad \sin\varphi = 0{,}447,\ \cos\varphi = 0{,}894$$

$$\text{and} \quad \underline{r} = 4\alpha \cdot \sin\varphi = \underline{1{,}79\,\alpha}$$

$$\tan\omega = \frac{1{,}5\alpha}{\alpha} = 1{,}5 \quad \Rightarrow \quad \omega = 56{,}31^{\circ}, \quad \sin\omega = 0{,}832, \quad \cos\omega = 0{,}555$$

$$\text{and} \quad \underline{\xi} = 4\alpha \cdot \sin\omega = \underline{3{,}33\alpha}$$

We can therefore write the three equations of equilibrium

$$\left(\sum M\right)_{\Delta}^{\curvearrowleft +} = 0 \;\Rightarrow\; 7\cdot 3\alpha - 2\cdot 2\alpha - 2\cdot\alpha - S_{14}\cdot 1{,}5a \;=\; 0$$

$$S_{14} \;=\; \frac{21-4-2}{1{,}5} \;=\; 10\,kN$$

$$\left(\sum M\right)_{\mathrm{E}}^{\curvearrowleft +} = 0 \;\Rightarrow\; 7\cdot 4\alpha - 2\cdot 3\alpha - 2\cdot 2\alpha - 2\cdot\alpha + S_{12}\cdot 1{,}79a \;=\; 0$$

$$S_{12} \;=\; -\frac{28-6-4-2}{1{,}79} \;=\; -8{,}94\,kN$$

$$\left(\sum M\right)_{A}^{\curvearrowleft +} = 0 \;\Rightarrow\; 2\cdot\alpha + 2\cdot 2a + 2\cdot 3\alpha + S_{13}\cdot 3{,}33a \;=\; 0$$

$$S_{13} \;=\; -\frac{2+4+6}{3{,}33} \;=\; -3{,}6\,kN$$

Now we can simply check the equations

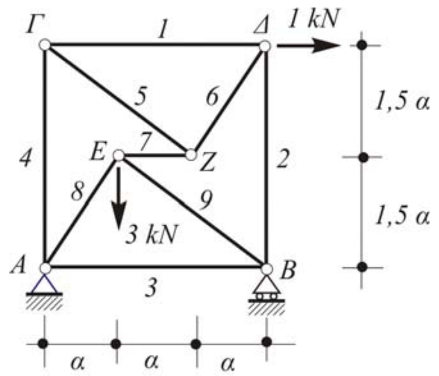
$$\sum x^{\overset{+}{\rightarrow}} = 0 \quad \text{and} \quad \sum y^{\uparrow +} = 0\,,$$

which of course we could use before to determine the forces S_{12} and S_{13}.

$$\sum x^{\overset{+}{\rightarrow}} \;=\; 10 - 3{,}6\cdot\cos\omega - 8{,}94\cdot\cos\varphi \;=\; 10-2-8 \;=\; 0!$$

$$\sum y^{\uparrow +} \;=\; 7-2-2-2-8{,}94\cdot\sin\varphi + 3{,}6\cdot\sin\omega \;=\; 1-3{,}99+2{,}99 \;=\; 0!$$

2 Face a graphical or analytical solution of the truss presented in the following figure.



Solution

The truss consists of two rigid triangles, ΓΔZ and ABE, connected through three members, 4, 7 and 2, which are not concurrent.

Obviously it is statically determinate and rigid, once the relation

$$m = 2j - 3 \qquad \text{or} \qquad 9 = 2\cdot 6 - 3$$

holds. The reactions can easily be calculated through the equilibrium equations:

$$(\Sigma M)_A^{\curvearrowleft +} = 0 \;\Rightarrow\; 1\cdot 3\alpha + 3\cdot a - B\cdot 3a \;=\; 0 \;\Rightarrow\; B = 2\,kN$$

$$\sum y^{\uparrow +} = 0 \;\Rightarrow\; A_y - 3 + 2 \;=\; 0 \;\Rightarrow\; A_y \;=\; 1\;kN$$

$$\sum x^{\overset{+}{\rightarrow}} \;=\; 0 \;\Rightarrow\; -A_x + 1 \;=\; 0 \;\Rightarrow\; A_x \;=\; 1\,kN$$

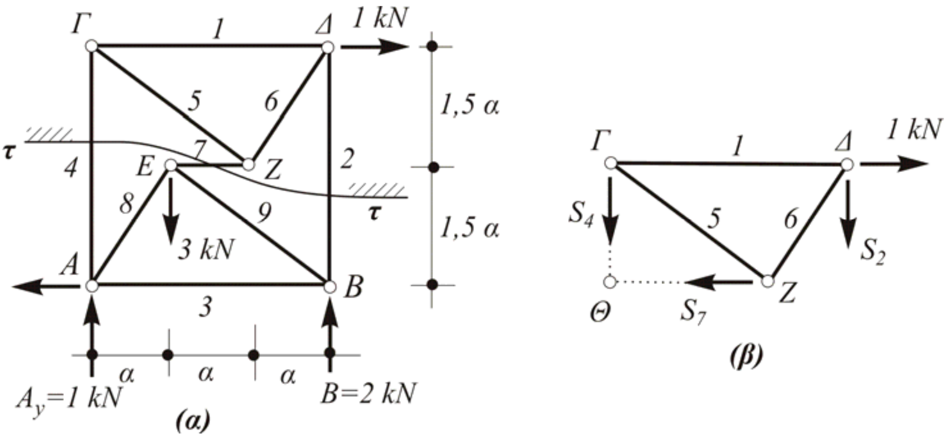
However, since on **all** joints concurrent are **3** unknown **forces**, it is **not** possible to apply neither the analytical method of joints nor the Cremona's graphical method.

Passing the section ττ, (figure β of the next page) we separate the upper portion, which is in equilibrium under the act of the forces 1 kN, S_2, S_4 and S_7.

Introducing all the unknown forces as tensile and writing the moment equation of equilibrium with respect to the point Θ it yields:

$$(\Sigma M)_{\Theta}^{\circlearrowleft +} = 0 \quad \Rightarrow \quad 1 \cdot 1.5\alpha + S_2 \cdot 3a \;=\; 0 \quad \Rightarrow \quad S_2 \;=\; -0.5 \;kN$$

Now writing the equations for vertical and horizontal equilibrium, we solve for the rest of the three unknown forces.



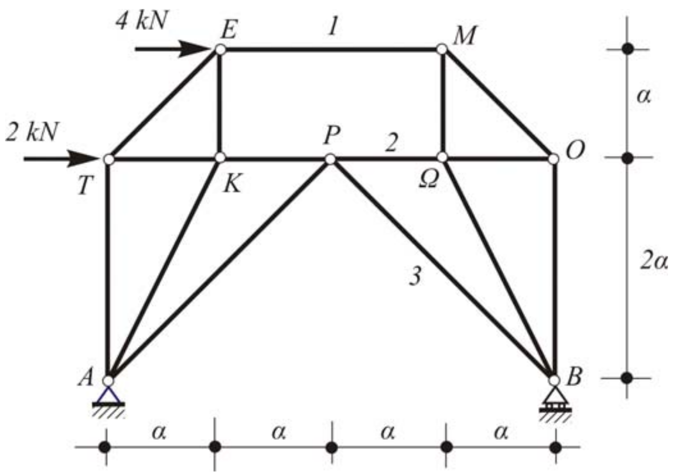
$$\sum y^{\uparrow +} \;=\; -S_4 + 0.5 \;=\; 0 \;\Rightarrow\; S_4 \;=\; 0.5 \;kN$$

$$\sum x^{\overset{+}{\rightarrow}} \;=\; 0 \;\Rightarrow\; 1 - S_7 \;=\; 0 \;\Rightarrow\; S_7 \;=\; 1\,kN\,.$$

After calculation of forces S_2, S_4 and S_7, (or only one of them) we can obviously continue with one of the preceding methods.

Note: The calculation of reactions here was not necessary. It was done either to use them for the lower portion, or to show the **impossible** of applying one of the conventional methods.

3 Face again a graphical or analytical solution of the truss presented in the following figure.



Solution

The truss is rigid once it consists of the rigid parts APKETA and BPΩMOB which are connected through the joint P and the member EM. Besides the relation

$$m = 2j - 3 \qquad \text{or} \qquad 15 = 2\cdot 9 - 3$$

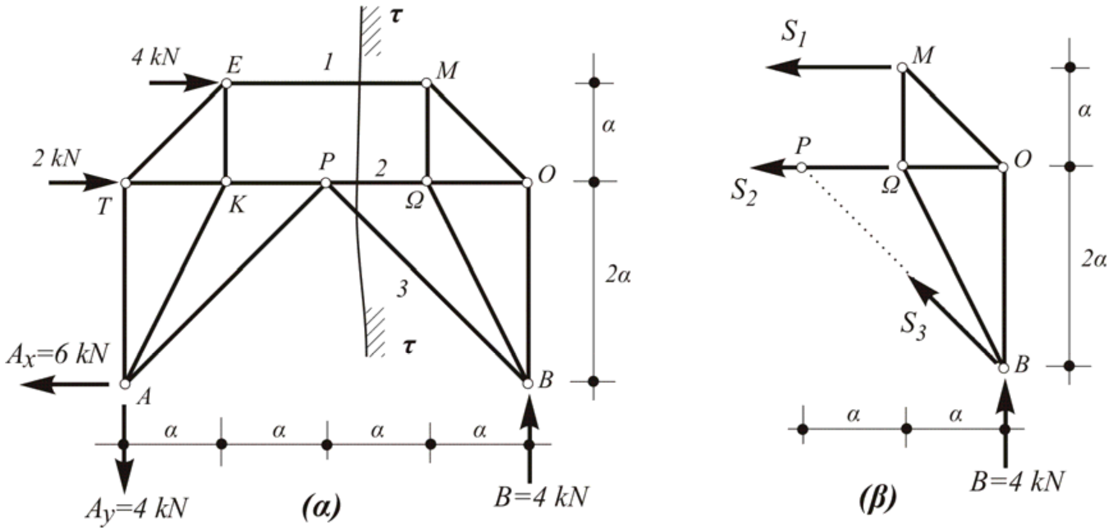
holds. As before, the reactions can easily be calculated through the equilibrium equations:

$$(\Sigma M)_A^{\circlearrowleft +} = 0 \Rightarrow 2\cdot 2\alpha + 4\cdot 3a - B\cdot 4a = 0 \Rightarrow B = 4\ kN$$

$$\sum y^{\uparrow +} = -A_y + 4 = 0 \Rightarrow A_y = 4\ kN$$

$$\sum x^{\overset{+}{\rightarrow}} = 0 \Rightarrow -A_x + 2 + 4 = 0 \Rightarrow A_x = 6\ kN .$$

Nevertheless, neither the method of joints nor the Cremona's diagram can be used to determine any member of the truss.



Passing the section ττ, which intersects members 1, 2 and 3, we separate the right portion BΩMOB (figure *β*), which is under the act of the reaction B = 4 kN and the forces S_1, S_2 and S_3. Introducing all the unknown forces as tensile and writing the moment equation of equilibrium with respect to the point P, it yields:

$$(\Sigma M)_P^{\circlearrowleft +} = 0 \Rightarrow -S_1\cdot\alpha - 4\cdot 2a = 0 \Rightarrow S_1 = -8\ kN$$

Now writing the equations for vertical and horizontal equilibrium, we solve for the rest of the three unknown forces.

$$\sum y^{\uparrow +} = 4 + S_3\cos 45^\circ = 0 \Rightarrow S_3 = -\frac{4}{0.707} = -5.657\ kN$$

$$\sum x^{\overset{+}{\rightarrow}} = 0 \Rightarrow 8 - S_2 + 5.657\cdot\cos 45^\circ = 0 \Rightarrow S_2 = 8 + 4 = 12\ kN .$$

Having determined S_3 or S_1, the calculation of the forces for the remaining members of the truss is possible if we start, by either method, from joint B or M respectively.