Exercise 1

An area of 10^4 km^2 of high seismicity is affected by an earthquake of magnitude M = 5.5 Richter every 80 years.

For this area the max acceleration is given through the following relationship:

$$\log a = 1.88 + 0.48M - 1.62\log(\Delta + 15)$$

where α is expressed in cm/sec^2 and Δ is the epicentral distance in km. Please:

1. Construct the probabilistic curve of exceeding (repeat period curve) in the seismic area. Before you draw the curve, calculate the repeat periods T_i, for the values of α_i = 50, 100, 150 and 200 cm/sec^2. Where is the curve converging, when T converges to infinite?
2. Calculate the design period T_d for a usual structure, when the probability of exceeding is p(Δt) = 0.10 and the useful life, Δt = 50 years. Which is then the corresponding peak ground acceleration?
3. Estimate the useful life that corresponds to the design – peak – ground – acceleration for a probability of exceeding p(Δt) = 0.20 and structures designed with an importance factor 1.30.

Solution

1. Probabilistic curve of exceeding

Taking into account the given data, for each acceleration value, we calculate (through the given relation) the epicentral distance Δ and then the repeat period T.

1. α_1 = 50 cm/sec^2

$$log50 = 1.88 + 0.48 \cdot 5.5 - 1.62log(\Delta_1 + 15)$$

$$lo\,g(\Delta_1 + 15) = 1.7414 \Rightarrow \Delta_1 + 15 = 10^{1.7414} \Rightarrow \Delta_1 \cong 40.13\,km$$

2. α_2 = 100 cm/sec^2

$$log100 = 1.88 + 0.48 \cdot 5.5 - 1.62log(\Delta_2 + 15)$$

$$lo\,g(\Delta_2 + 15) = 1.5556 \Rightarrow \Delta_2 + 15 = 10^{1.5556} \Rightarrow \Delta_2 \cong 20.94\,km$$

3. α_3 = 150 cm/sec^2

$$log150 = 1.88 + 0.48 \cdot 5.5 - 1.62log(\Delta_3 + 15)$$

$$lo\,g(\Delta_3 + 15) = 1.4469 \Rightarrow \Delta_3 + 15 = 10^{1.4469} \Rightarrow \Delta_3 \cong 12.98\,km$$

4. α_4 = 200 cm/sec^2

$$log200 = 1.88 + 0.48 \cdot 5.5 - 1.62log(\Delta_4 + 15)$$

$$lo\,g(\Delta_4 + 15) = 1.3697 \Rightarrow \Delta_3 + 15 = 10^{1.3697} \Rightarrow \Delta_3 \cong 8.43\,km$$

The relation that compares the repeat periods T_0 and T_i with the areas A_0 and A_i for two different regions under the same seismic event is:

$$\frac{A_0}{A_i} = \frac{T_i}{T_0}$$

where: $A_i = \pi\Delta_i^2$, $A_0 = 10^4$ km^2 and T_0 = 80 years. Therefore

$$T_i = \frac{A_0 \cdot T_0}{\pi {\Delta_i}^2}$$

For Δ_1 = 40.13 km

$$T_1 = \frac{10^4 \cdot 80}{\pi \cdot 40.13^2} = 158.14\, years$$

For Δ_2 = 20.94 km

$$T_2 = \frac{10^4 \cdot 80}{\pi \cdot 20.94^2} = 580.85\, years$$

For Δ_3 = 12.98 km

$$T_3 = \frac{10^4 \cdot 80}{\pi \cdot 12.98^2} = 1511.30\, years$$

For Δ_4 = 8.43 km

$$T_4 = \frac{10^4 \cdot 80}{\pi \cdot 8.43^2} = 3585.05\, years$$

The above results are tabled as follows:

Acceleration (cm/sec^2)	Epicentral distance (km)	Repeat period (years)
50	40.13	158.14
100	20.94	580.85
150	12.98	1511.30
200	8.43	3585.05

On the basis of these data the repeat-period-curve can be constructed. In fact, through a brief **EXCEL** program developed for this purpose, many different values of the above parameters can be provided showing thus the change between acceleration and repeat period, of course through the epicentral distance which does not appear in the graph. **The program data along with the graph are depicted on the last page.**

When T converges to infinite, obviously the acceleration converges to a maximum value which corresponds to a zero epicentral distance.

This maximum value can be estimated from the initial formula, putting Δ = 0.

$$\log a_{max} = 1.88 + 0.48 \cdot 5.5 - 1.62 \log(0 + 15)$$

$$\Rightarrow log a_{max} = 2.6147 \quad \Rightarrow \quad \boldsymbol{a_{max} = 411.84\, cm/sec^2}$$

2. Design period

Given are: Δt = 50 years and $p(\Delta t)$ = 0.10. The design period is therefore:

$$\boldsymbol{T_d} = \frac{-\Delta t}{\ln(1-p)} = \frac{-50}{\ln(1-0.1)} = \boldsymbol{474.56\ years}$$

The peak ground acceleration can be approached by two ways:

i) Directly through the probabilistic curve of exceeding and
ii) Following the reverse procedure; i.e. from T_d (years), estimating the epicentral distance Δ, which then yields α.

In our case, for T_i =474.56 years, the previous relation,

$$T_i = \frac{A_0 \cdot T_0}{\pi {\Delta_i}^2} \quad \rightarrow \quad 474.56 = \frac{10^4 \cdot 80}{\pi {\Delta_i}^2}$$

yields Δ_i = 23.16 km. Then from the initial formula we get:

$$\log \alpha = 1.88 + 0.48 \cdot 5.5 - 1.62 \log(23.16 + 15), \text{ or}$$

$\log\alpha$ = 1.9578 and **α = 90.74 cm/sec^2**.

3. Useful life

Given are: Importance factor = 1.30 and $p(\Delta t)$ = 0.20.

Once the importance factor affects the design peak ground acceleration α_d, the resulting new one is: $\alpha_d = \Sigma 3 \cdot \alpha = 1.30 \cdot 90.74 = 117.96$ cm/sec^2.

Then the equation

$$\log 117.96 = 1.88 + 0.48 \cdot 5.5 - 1.62 \log(\Delta + 15)$$

will yield the value of the epicentral distance Δ = 17.45 km.

The corresponding design period is therefore:

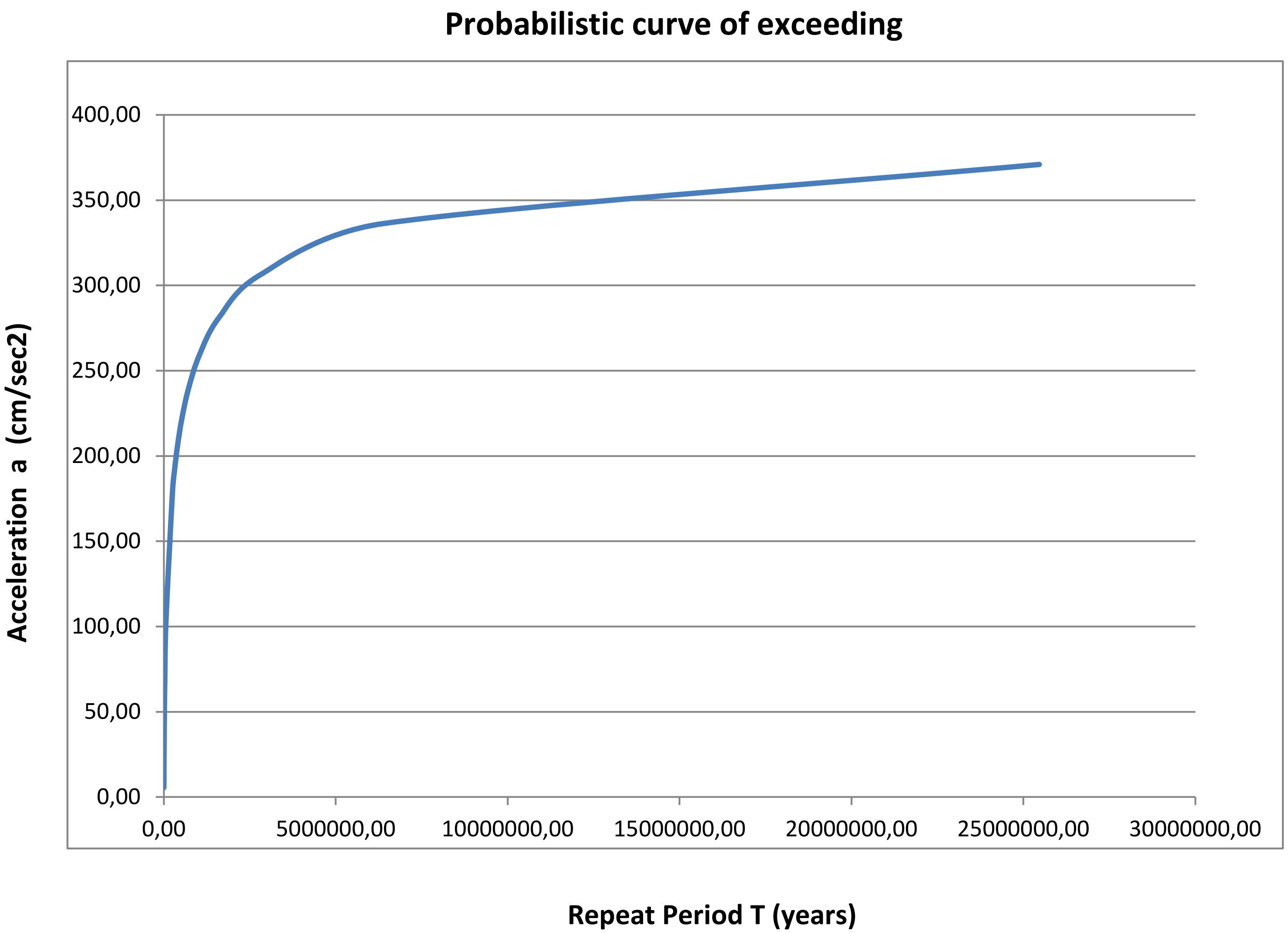
$$T_{new} = \frac{A_0 \cdot T_0}{\pi {\Delta_i}^2} = \frac{10^4 \cdot 80}{\pi \cdot 17.45^2} = 835.85\ years$$

Finally, the life period is estimated from the formula:

$$T_d = \frac{-\Delta t}{\ln(1-p)} \Rightarrow 835.85 = \frac{-\Delta t}{\ln(1-0.2)}$$

$\Rightarrow$ - Δt = 835.85·ln0.8 and **Δt = 189 years**.

Repeat Period T (years)	Acceleretion a (cm/sec2)	Epicntrl Dist (km)	loga
101859249,62	390,54	0,5	2,5917
25464812,40	370,96	1	2,5693
6366203,10	336,26	2	2,5267
2829423,60	306,52	3	2,4865
1591550,78	280,81	4	2,4484
1018592,50	258,42	5	2,4123
707355,90	238,78	6	2,3780
519690,05	221,45	7	2,3453
397887,69	206,06	8	2,3140
314380,40	192,33	9	2,2841
254648,12	180,03	10	2,2553
63662,03	104,38	20	2,0186
28294,24	69,47	30	1,8418
15915,51	50,19	40	1,7006
10185,92	38,29	50	1,5831
7073,56	30,37	60	1,4824
5196,90	24,79	70	1,3943
3978,88	20,71	80	1,3161
3143,80	17,61	90	1,2457
2546,48	15,19	100	1,1817
636,62	5,51	200	0,7414



Exercise 2

An earthquake of magnitude 6.0 on the Richter scale occurs every 90 years in a region of 10^4 km^2, where a structure is going to be constructed.

Following the directions provided in the Greek Seismic Code, for q=1 and soil class A,

1. Calculate the peak ground acceleration expected at the site of the structure for the above earthquake.
2. Draw the probabilistic curve of exceeding for the peak ground acceleration at the site of the structure.
3. Draw the elastic design acceleration spectrum
   a) For 20% probability of exceeding in 50 years and
   b) For 10% probability of exceeding in 80 years,

Recommendation: Use the Ambrasseys, Simpson & Bommer (1996) attenuation relationship for rocks and 16% probability of exceeding.

Solution

The expected peak ground acceleration for an earthquake **at the site** of the structure can be calculated through the attenuation relationship of Ambraseys, Simpson & Bommer (1996) for **Δ=0**.

$$\log\alpha = -1.47 + 0.266M - 0.922\cdot\log R + 0.100S_A + 0.094S_S + 0.25P,$$

where $M=6.0$, $R = \sqrt{\Delta^2 + 3.5^2}$ and $\Delta=0$.

$S_{A,S}$ are dummy variables for the site class. For rocks, their values are:

$$S_A = 0 \text{ and } S_S = 0.$$

It is also P=1 for 16% probability of exceeding. Therefore:

$$\log\alpha = -1.47 + 0.266x6.0 - 0.922\log(3.5) + 0.25, \text{ or}$$

$\log\alpha = -0.1256$ and finally $\boldsymbol{\alpha = 0.749}$.

1. In order to draw the probabilistic curve of exceeding, which represents the values of ground acceleration versus repeat period, we have to fill up the following table:

α_i(g)	Δ_i(km)	T_i(years)
0.05	65.81	66.15
0.10	30.88	300.48
0.20	14.23	1415.10
0.30	8.77	3728.09
0.40	5.96	8073.45

0.50	4.14	16685.90
0.70	1.39	148548.07
0.74	0.56	899710.03

Example of procedure:

For $\alpha_i = 0.10$,

$$\log 0.10 = -1.47 + 0.266 \cdot 6.0 - 0.922 \cdot \log\sqrt{\Delta_i^2 + 3.5^2} + 0.25, \quad \text{or}$$

$$\log\sqrt{\Delta_i^2 + 3.5^2} = 1.492, \quad \text{or} \quad \Delta_i = 30.88 \text{ km}.$$

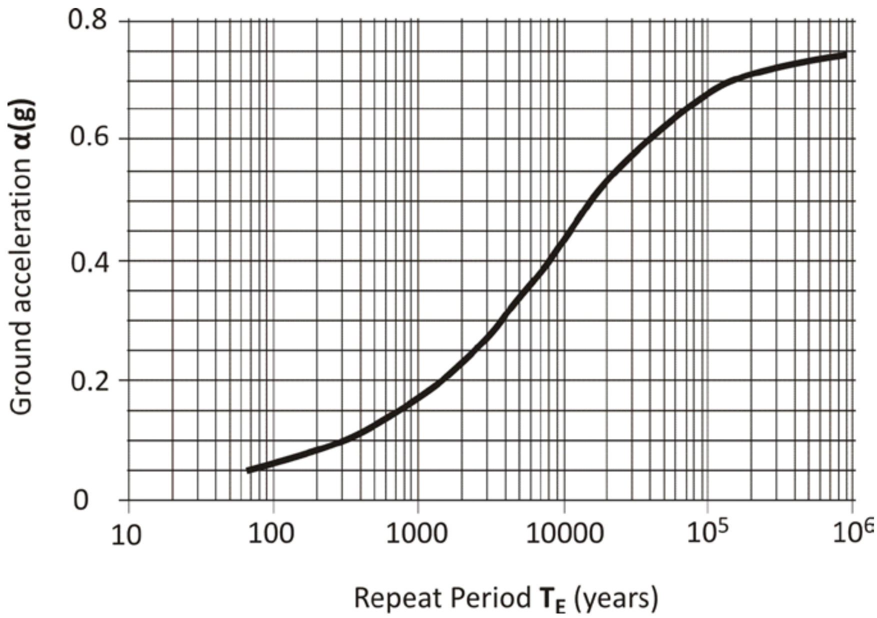
Then, taking into account that the seismic area A_i (associated with the value a_i) is a circle with a radius Δ_i, where the unknown repeat period T_i corresponds, from the data of the given area A_0 with its repeat period T_0, we can solve for T_i the equation

$$A_0 T_0 = A_i T_i \quad \text{where} \quad A_i = \pi \cdot \Delta_i^{\ 2}$$

Therefore: $T_i = A_0 T_0 / \pi \Delta_i^2$.

If we put: $A_0 = 10^4$ km, $T_0 = 90$ years and $\Delta_i = 30.88$ km, then it yields

$T_i = 10^4 \text{x} 90/\pi \cdot 30.88^2 = 300.48$ years.



2. The relation connecting the repeat period, T_E, of an earthquake along with a structure's useful period of life, Δt, and the probability p of exceeding the earthquake's magnitude, is

$$T_E = \frac{-\Delta t}{\ln(1-p)}$$

a) If we put $\Delta t = 50$ and $p = 0.20$, it yields $T_E = 224.07$ years

Using the above graph, referring to the probabilistic curve of exceeding, for $T_E = 224.07$ years, we end up with the corresponding peak ground acceleration, which is $a_a = 0.07g$.

b) Similarly, in the above equation, if we put $\Delta t = 80$ and $p = 0.10$, it yields $T_E = 759.3$ years.

Again, making use of the above graph, for $T_E = 759.3$ years, we find the corresponding acceleration $a_b = 0.15g$.

For both cases, the elastic design acceleration spectrum will be created according to the Greek seismic code, taking into account the restrictions of the problem.

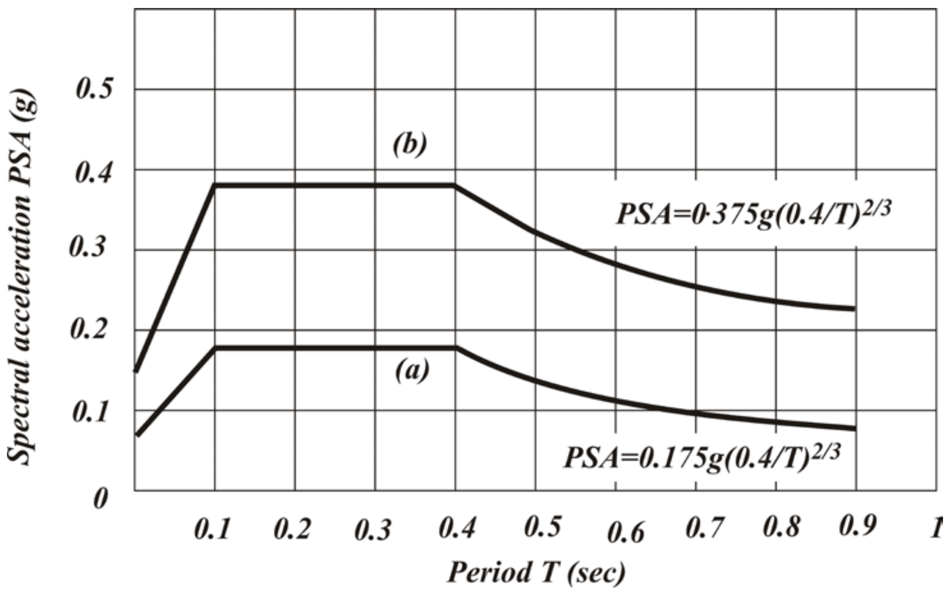
For soil class A, it is $T_1 = 0.1$ and $T_2 = 0.4$.

Also for $q = 1$, it is:

$$\frac{\Phi_d(T)}{A\gamma_I} = \frac{\eta\theta\beta_0}{q} \quad or \quad \frac{\Phi_d(T)}{A \cdot 1} = \frac{1 \cdot 1 \cdot 2.5}{1} = 2.5$$

Therefore: $\Phi_d(T_a)(g) = 2.5 \cdot 0.07 = 0.175$ and

$\Phi_d(T_b)(g) = 2.5 \cdot 0.15 = 0.375$



The spectrum starts from the peak ground acceleration and increases linearly up to the point $[0.1, \Phi_d(T_1)]$, because $T_1 = 0.1$.

Then, keeping a constant value up to $T_2 = 0.4$, it follows the path shown by the corresponding equation, in which equation the first term is the constant value $\Phi_d(T_1)$ of the spectral acceleration, while the second term is the ratio T_2/T, raised to the power of 2/3.

Exercise 3

A research for determining the local seismic hazard where an important structure is going to be constructed, gave the curve shown in Fig. 1. **Calculate**:

1. The peak ground acceleration, according to which common structures are going to be constructed for a life duration and a probability of exceeding proposed in EC8.
2. The life duration, corresponding to the peak ground acceleration, for a structure of important factor 1.15 and a probability of exceeding 20%.
3. The max magnitude of an earthquake, assuming that its epicenter is located at the site of the structure. Use Fig.1 and the following attenuation relationship:

$\log A = 1.86 + 0.49M - 1.65 \log(\Delta+15)$ (Δ in km, A in cm/sec^2, and g = 10 m/sec^2)

An earthquake in the area of the structure resulted to the spectrum shown in Fig. 2. Calculate the probability of occurrence for a life duration 50 years.

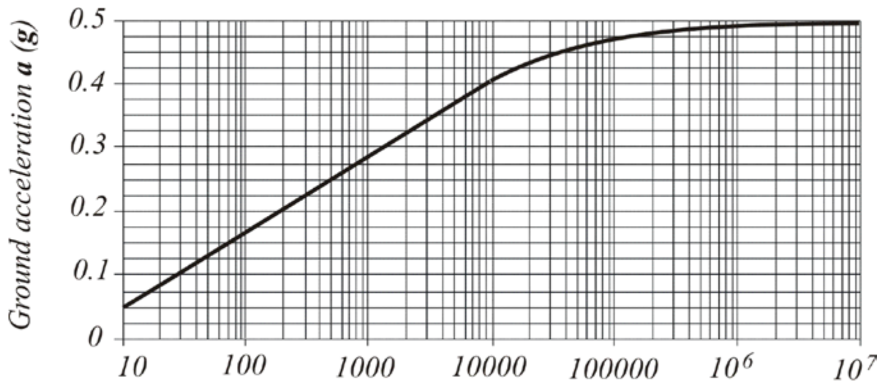


Figure 1

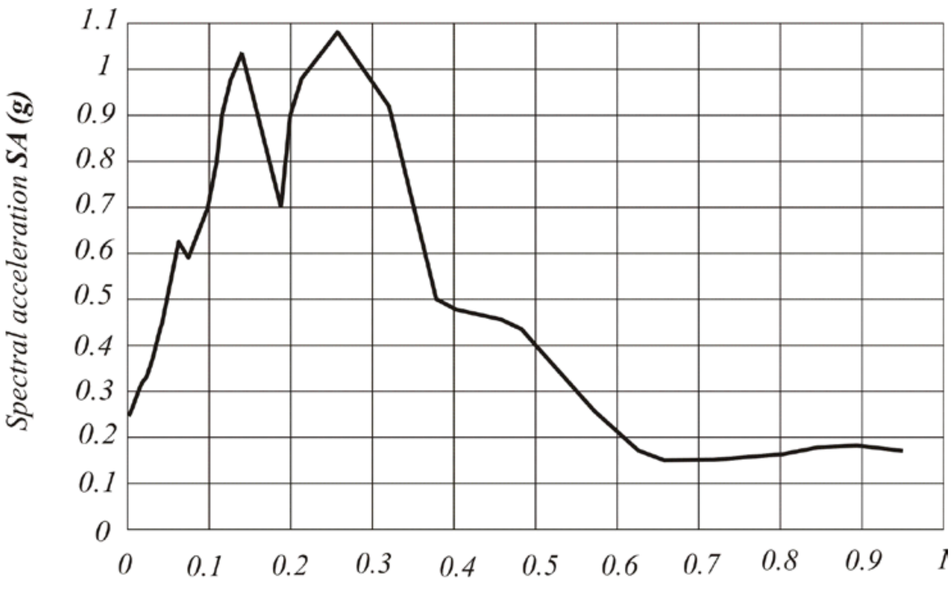


Figure 2

Solution

According to EC8, the proposed life duration for **common** structures is Δt = 50 years, while the probability of exceeding is p = 10%.

The repeat period is therefore:

$$T_E = \frac{-\Delta t}{\ln(1-p)} = \frac{-50}{ln0.90} = 475\, years$$

Using the curve in Fig. 1 for the above repeat period, we find the corresponding peak ground acceleration **A = 0.24g**.

1. The design acceleration, being dependant on the importance factor, is for the new building, A = 1.15·0.24g = 0.276g.

Using the same curve for the new ground acceleration, we find T_E = 800 years. Therefore:

$$T_E = \frac{-\Delta t}{\ln(1-p)} \quad \Rightarrow \quad 800 = \frac{-\Delta t}{ln0.90} \quad \Rightarrow \quad \boldsymbol{\Delta t = 84\, years}$$

2. Since the epicenter of the earthquake is on the site of the structure, it follows that Δ = 0.

The **max** acceleration, according to fig. 1, is: maxA = 0.5g = 500 cm/sec^2.

Therefore, the attenuation relationship becomes

log500 = 1.86 + 0.49M – 1.65·log(0 + 15),

from which, the yielding max magnitude of the earthquake, is **M = 5.7**

3. Using the spectrum of fig. 2, **for T = 0**, we obtain the peak ground acceleration, A = 0.25g.

From the curve of Fig. 1, for A = 0.25g, we find T_E = 500 years.

Therefore:

$$T_E = \frac{-\Delta t}{\ln(1-p)} \quad \Rightarrow \quad 500 = \frac{-80}{\ln(1-p)} \quad \Rightarrow \quad \ln(1-p) = -0.16$$

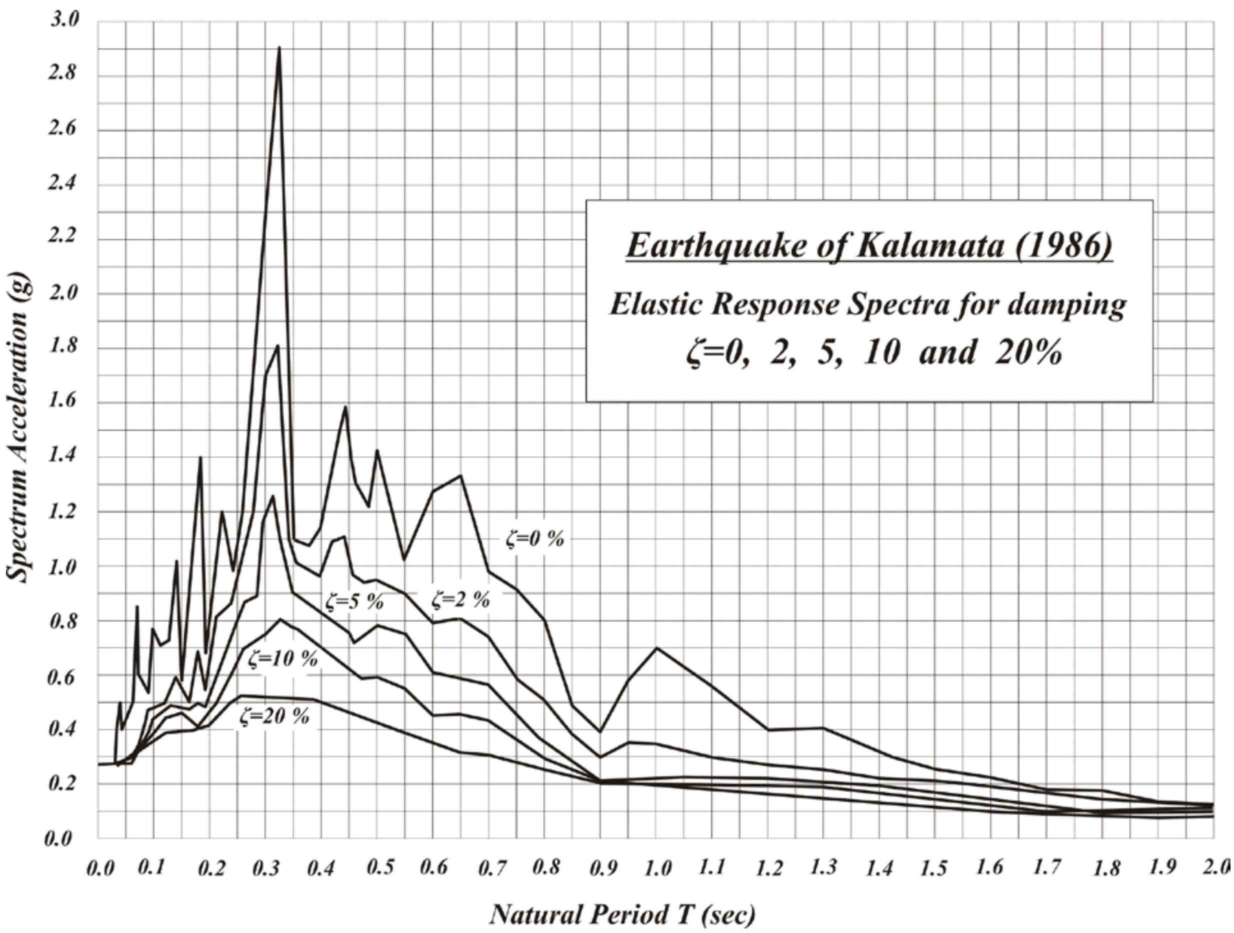
Consequently

$$1 - p = e^{-0.16} = 0.852 \quad \Rightarrow \quad \mathbf{p} = 0.148 = \mathbf{14.8\,\%}.$$

Exercise 4

A series of elastic acceleration spectra of Kalamata's earthquake (1986) is depicted in the figure below. Calculate:

1. The max acceleration of the earthquake.
2. The spectral magnification factor, i.e. the ratio of max spectral acceleration to the max ground acceleration, for dumping ratios $\zeta = 2$, 5 and 10 %.
3. The max displacement and the max seismic force of the following structures, which present the following respective seismic characteristics:
   a) Reinforced concrete (RC) building: T = 0.12 sec, m = 1000 t and $\zeta = 5$ %
   b) R.C. building: T = 0.25 sec, m = 3000 t and $\zeta = 5$ %
   c) R.C. structure: T = 0.32 sec, m = 3000 t and $\zeta = 5$ %
   d) R.C. bridge: T = 1.20 sec, m = 10000 t and $\zeta = 5$ %
   e) Steel structure: T = 0.60 sec, m = 500 t and $\zeta = 2$ % and
   f) Timber building: T = 0.20 sec, m = 200 t and $\zeta = 10$ %.
4. Draw the relative displacement-spectrum for $\zeta = 5$ %. Calculate the values of displacement by taking natural periods from 0 to 1.0, using a step of 0.10 sec.



Solution

1. The max acceleration of the earthquake corresponds to T = 0, when the structure obviously cannot undertake any relative displacement.

From the spectrum, for T = 0 yields **PSA = 0.27g**.

2. The maximum spectrum accelerations, corresponding to the three requested damping ratios are:

For ζ = 2 % $\Rightarrow$ PSA = 1.82 g

For ζ = 5 % $\Rightarrow$ PSA = 1.25 g

For ζ = 10 % $\Rightarrow$ PSA = 0.80 g

Consequently the spectral magnification factor is respectively:

For ζ = 2 % $\Rightarrow$ $\beta = \frac{1.82 \cdot g}{0.27 \cdot g} = \mathbf{6.74}$

For ζ = 5 % $\Rightarrow$ $\beta = \frac{1.25 \cdot g}{0.27 \cdot g} = \mathbf{4.63}$

For ζ = 10 % $\Rightarrow$ $\beta = \frac{0.80 \cdot g}{0.27 \cdot g} = \mathbf{2.96}$

3. Maximum displacement and seismic force
   a) R.C. building, T = 0.12 sec, m = 1000 t and ζ = 5 %

From spectrum, for T = 0.12 $\Rightarrow$ PSA = 0.45 g

The max seismic force is estimated through the equation P = PSA·m, where m is the mass of the structure. Hence:

$\mathbf{P_{max}}$ = 0.45·g·1000 t = 1000 Mgr·0.45·10 m/sec^2 = **4500 kN**.

From theory of single degree of freedom (SDOF) structures, it holds:

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \Rightarrow \quad k = \frac{4\pi^2 m}{T^2}$$

where k is the stiffness of the structure. If δ is the displacement of the above mass, then P = k·δ. In this equation, if k is replaced by the value taken from the previous equation, it yields

$$P = \frac{4\pi^2 m}{T^2}\delta = PSA \cdot m \quad \Rightarrow \quad \delta = \frac{PSA \cdot T^2}{4\pi^2}$$

The displacement, δ, of the structure is therefore:

$$\boldsymbol{\delta} = \frac{0.45 \cdot 10 \frac{m}{sec^2} \cdot 0.12^2 sec^2}{4\pi^2} = \mathbf{0.0016\,m}$$

b) R.C. building: T = 0.25 sec, m = 3000 t and ζ = 5 %

Similarly, from spectrum, for T = 0.25 ⇒ PSA = 0.80 g. Therefore

P_{max} = 3000·0.80·10 = **24000 kN** and

$$\boldsymbol{\delta} = \frac{PSA \cdot T^2}{4\pi^2} = \frac{0.80 \cdot 10 \cdot 0.25^2}{4\pi^2} = \mathbf{0.013\ m}$$

c) R.C. structure: T = 0.32 sec, m = 3000 t and ζ = 5 %

Similarly, from spectrum, for T = 0.32 ⇒ PSA = 1.25 g. Therefore

P_{max} = 3000·1.25·10 = **37500 kN** and

$$\boldsymbol{\delta} = \frac{PSA \cdot T^2}{4\pi^2} = \frac{1.25 \cdot 10 \cdot 0.32^2}{4\pi^2} = \mathbf{0.033\ m}.$$

d) R.C. bridge: T = 1.20 sec, m = 10000 t and ζ = 5 %

Similarly, from spectrum, for T = 1.20 ⇒ PSA = 0.25 g. Therefore

P_{max} = 10000·0.25·10 = **25000 kN** and

$$\boldsymbol{\delta} = \frac{PSA \cdot T^2}{4\pi^2} = \frac{0.25 \cdot 10 \cdot 1.20^2}{4\pi^2} = \mathbf{0.091\ m}$$

e) Steel structure: T = 0.60 sec, m = 500 t and ζ = 2 %

Similarly, from spectrum, for T = 0.60 and ζ = 2% ⇒ PSA = 0.80 g. Therefore

P_{max} = 500·0.80·10 = **4000 kN** and

$$\boldsymbol{\delta} = \frac{PSA \cdot T^2}{4\pi^2} = \frac{0.80 \cdot 10 \cdot 0.60^2}{4\pi^2} = \mathbf{0.073\ m}$$

f) Timber building: T = 0.20 sec, m = 200 t and ζ = 10 %

Similarly, from spectrum, for T = 0.20 and ζ = 10% ⇒ PSA = 0.45 g. Therefore

P_{max} = 200·0.45·10 = **900 kN** and

$$\boldsymbol{\delta} = \frac{PSA \cdot T^2}{4\pi^2} = \frac{0.45 \cdot 10 \cdot 0.20^2}{4\pi^2} = \mathbf{0.0045\ m}$$

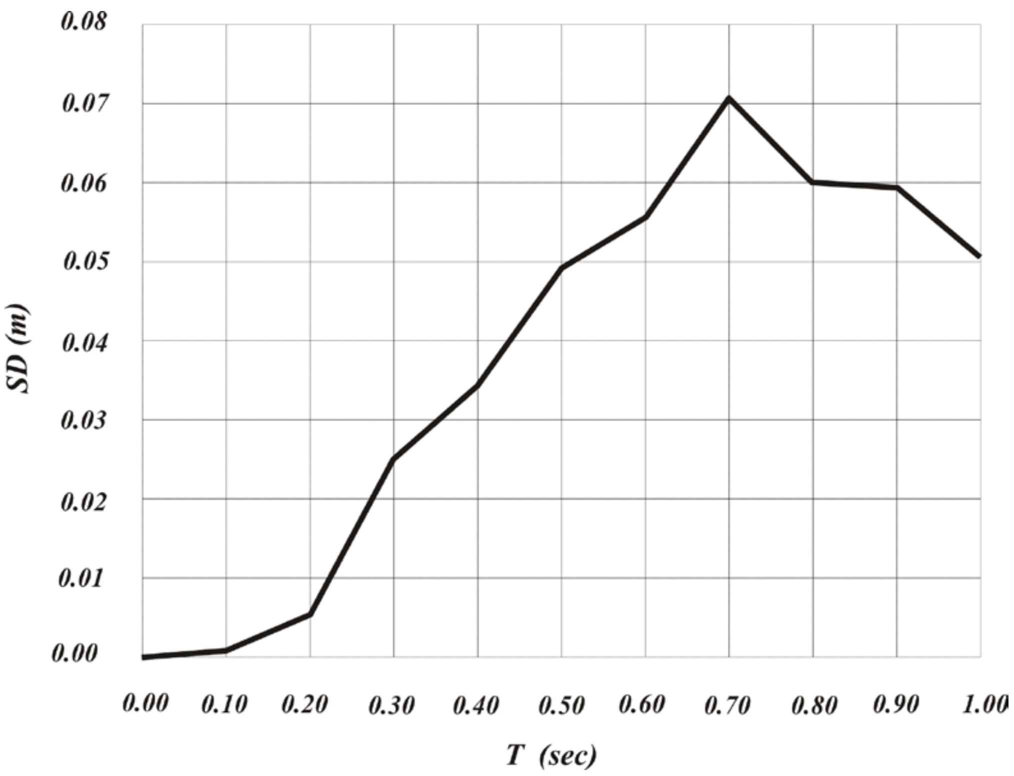
4. Displacement spectrum

The displacements SD will be calculated through the equation SD = PSA/ω^2, where:

$$\omega = \frac{2\pi}{T}$$

For each value of period T, a value of acceleration is yielded through the spectrum, along with a value of ω. The following table summarizes the results.

T (sec)	PSA (m/sec^2)	ω (1/sec)	SD (m)
0.00	2.70	∞	0.0000
0.10	4.00	62.83	0.0010
0.20	5.50	31.42	0.0056
0.30	11.20	20.94	0.0251
0.40	8.40	15.71	0.0340
0.50	7.70	12.57	0.0488
0.60	6.10	10.47	0.0556
0.70	5.70	8.98	0.0707
0.80	3.70	7.85	0.0600
0.90	2.90	6.98	0.0595
1.00	2.00	6.28	0.0507



Displacement spectrum

Exercise 5

Two similar water towers, illustrated on Fig. 1 of next page, are founded on different grounds; one on the rock at point A, the other on a thick ground layer at point B.

During a seismic event, two accelerographs, that existed on places A and B, recorded this vibration. The data analysis of records which followed, gave the elastic spectral accelerations (damping ratios ζ = 5%), depicted on Figure 2.

Calculate:

1. The max acceleration developed on the base of each tower.
2. The max acceleration and the corresponding seismic force developed on the center of gravity (CG) of each tower.
3. The shear force and bending moment developed on the base of each column, provided the structure behaved elastically.
4. The max elongation of water pipe that connects the two towers between the points A and B.
5. The max elongation of the same water pipe, if it connected the two towers between the points A' and B'.
6. Estimate the dumping ratio ζ of the thick ground layer, considering that it behaves elastically.

Data – Assumptions:

- The water towers present a dumping ratio ζ = 5%, which is different from that of the ground layer.
- The towers rest on 4 similar columns, having a cross section 0.50 x 0.50 m and a height h = 6.0 m.
- Total weight of each tower, included water, is W = 1000 kN.
- Young modulus of elasticity for Reinforce Concrete, $E = 21\cdot10^3$ MPa.
- The ground layer behaves as SDOF with a self period, Tg = 0.5 sec.
- The points A and B'' of the rock move together as a unit.
- Before calculating the dumping ratio ζ of the ground layer, take into account the modification factor η, given by the Greek Seismic Code (EAK 2000), where it is stated that

$$PSA(\zeta) = PSA(\zeta{=}5\%)\cdot\eta ,$$

where

$$\eta = \sqrt{\frac{7}{\zeta + 2}} .$$

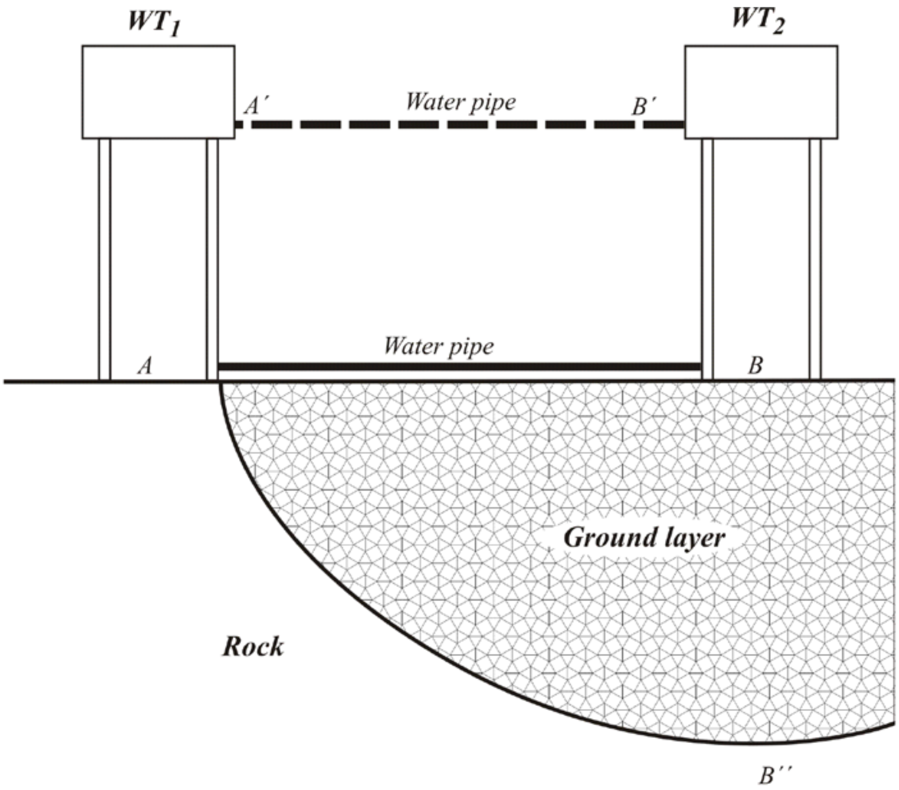


Figure 1

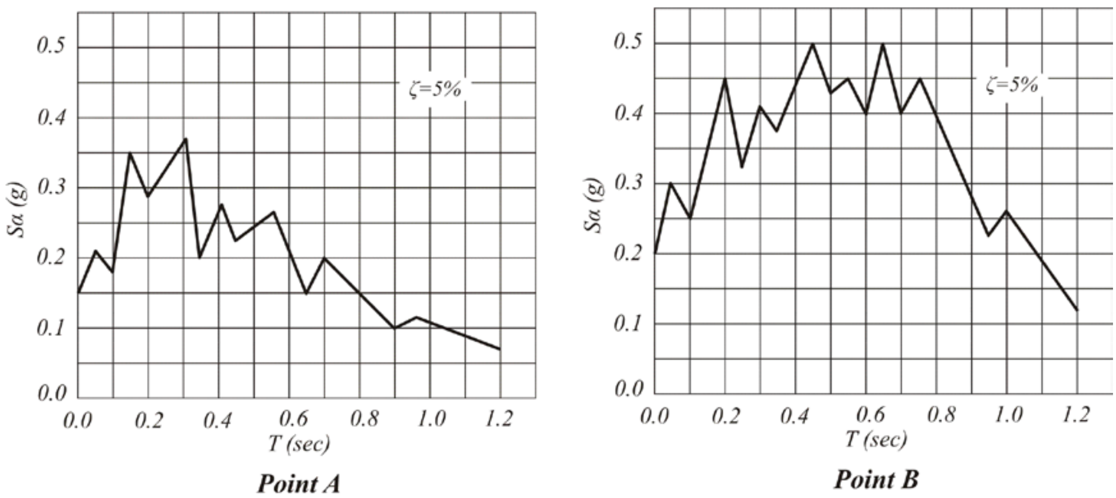


Figure 2

Solution

Using the elastic acceleration spectra for points A and B for T = 0, we get directly the max ground acceleration:

For point A: Sα (A) = 0.15g

For point B: Sα (B) = 0.20g

1. Since the dumping ratios for both – the spectra and towers – are the same, i.e. ζ = 5%, the max acceleration, developed on the center of gravity (CG) of each water tower, is possible to be estimated through their self-period T, making use of their elastic acceleration spectra. If T_t is the self-period of the water tower, it is:

$$T_t = 2\pi \sqrt{\frac{m}{k_{tot}}}$$

where m is the total tower mass which is:

m = W/g = 1000 kN/10 m/sec^2 = **100 kN·m^{-1}·sec^2**

and k_{tot} is the total tower stiffness, which is: $k_{tot} = 4k_c$. A double fixed column, obviously develops a stiffness, k_c, which is:

$$k_c = \frac{12EJ}{h^3}$$

where E is the Young modulus of elasticity, h the height of column and J the second moment of area of its cross section, which is:

$$J = \frac{bh^3}{12} = \frac{0.50^4}{12} = 0.005208\ m^4$$

Therefore

$$k_c = \frac{12 \cdot 21 \cdot 10^6 \frac{kN}{m^2} \cdot 0.005208 m^4}{6.0^3\ m^3} = 6076\ kN/m$$

which yields a $\mathbf{k_{tot}}$ = $4k_c$ = 4·6076 = **24305 kN/m** and finally a self-period of tower

$$T = 2\pi \sqrt{\frac{100\ kN{\cdot}m^{-1}{\cdot}sec^2}{24305\ kN/m}} = 0.403 \text{ sec}$$

For the water tower WT_1, the spectrum at point A for a period T = 0.403 sec, gives an acceleration of:

Sα (A) = 0.275g

Similarly, for the tower WT_2, the spectrum at point B for the same period T = 0.403 sec, gives an acceleration of:

$$S\alpha\ (B) = 0.45g.$$

Therefore the horizontal seismic forces developed at their center of gravity are:

$$F_1 = m \cdot S\alpha(A) = 100\ kN \cdot m^{-1} \cdot sec^2 \cdot 0.275g = 275\ kN \text{ and}$$

$$F_2 = m \cdot S\alpha(B) = 100\ kN \cdot m^{-1} \cdot sec^2 \cdot 0.45g = 450\ kN.$$

2. The maximum shear force developed at the base of each column, Q, is the quarter of the corresponding force acted at the CG. Furthermore, for a double fixed column, the bending moment at both, foot and head sections, is $M = Q \cdot h/2$. Hence:

For water tower WT_1: $Q_1 = F_1/4 = 275/4 = 68.75$ kN and

$M_1 = Q_1 \cdot h/2 = 68.75 \cdot 6/2 = 206.25$ kNm, while

For water tower WT_1: $Q_2 = F_2/4 = 450/4 = 112.5$ kN and

$M_2 = Q_2 \cdot h/2 = 112.5 \cdot 6/2 = 337.5$ kNm.

3. The maximum elongation of the water pipe is obviously expressed by the relevant displacement of the point A with respect to B.

Point A is on the rock while point B is on a ground layer founded on the rocky mass. Besides given is that the rocky mass is moving as a solid body, i.e. points A and B'' move in the same way. Therefore the problem is to find out the **relevant displacement of point B'' with respect to B**.

It has been assumed that the ground layer behaves as SDOF oscillator founded on the rock, with a self-period $T_g = 0.5$ sec. For every SDOF oscillator is stated that:

$$Sd = \frac{Sa}{\omega^2} = \frac{Sa}{\left(\frac{2\pi}{T}\right)^2}$$

where: Sd is the spectral relevant displacement of the oscillator
Sα is its absolute acceleration and
T, ω are respectively its natural period and frequency.

In our case Sα is the absolute spectral ground-layer-mass acceleration (the mass is considered to be concentrated in the point B), where it is $S\alpha(B) = 0.20g$. Therefore:

$$Sd(B) = \frac{S\alpha(B)}{\left(\frac{2\pi}{T_g}\right)^2} = \frac{0.20g}{\left(\frac{2\pi}{0.5\ sec}\right)^2} = 0.012\ m$$

In other words, since point B (ground) has been moved with respect to point B'' (rock) **0.012 m**, this distance obviously represents the **max elongation** of the water pipe between the points A and B.

4. For two oscillators, presenting self-periods T_1, T_2 and dumping ratios ζ_1, ζ_2 their max distance is

$$\Delta l = \sqrt{u_1^2 + u_2^2}\,,$$

where u_1, u_2 is the max displacement of each oscillator **with respect to its base**.

Regarding the case of relevant displacements, Sd(A′), Sd(B′) of the water towers with respect to their base, it holds that:

$$Sd(A') = \frac{S\alpha(A')}{\left(\frac{2\pi}{T}\right)^2} = \frac{0.275g}{\left(\frac{2\pi}{0.403}\right)^2} = 0.011\,m$$

$$Sd(B') = \frac{S\alpha(B')}{\left(\frac{2\pi}{T}\right)^2} = \frac{0.45g}{\left(\frac{2\pi}{0.403}\right)^2} = 0.018\,m$$

The max elongation of the water pipe A′B′ is therefore

$$\Delta l = \sqrt{0.011^2 + 0.018^2} = 0.021\,m$$

5. The ground layer is simulated with a SDOF oscillator presenting a self-period T = 0.5 sec, which, having fixed (founded) on point B′′ of the rock, has a maximum acceleration obtained from the spectrum, PSα(**B**) = 0.20g. Since the rocky mass is moving as a solid body (points A and B′′ present the same displacement), it yields that

Spectrum of point **B′′** = Spectrum of point **A**

The dumping ratio ζ of the ground layer is unknown; however, if it was 5%, like rock's, then we could use for the ground the spectrum of point A.

Using the spectrum of point A, for T_g = 0.5 sec, we find a max acceleration for an **imaginary** point **B** in the case where all the ground was a rock, PSα(**B**)[**5%**] = 0.25g, which is different from the **real** one, PSα(**B**)[**ζ**] = 0.20g. The difference of the two acceleration values is due to the different dumping of the ground layer.

From the Greek Seismic Code, it holds:

PSα(B)**[ζ] =** PSα(B)**[5%]·η** (1) , where $\eta = \sqrt{\frac{7}{\zeta+2}}$ (2) .

From (1) ⇒ 0.20g = 0.25g·η ⇒ η = 0.8.

From (2) ⇒ $0.8 = \sqrt{\frac{7}{\zeta+2}}$ ⇒ **ζ = 8.94%.**

Exercise 6

A) The single-storey R.C. structure illustrated below, was designed and constructed following the terms of the Greek Seismic Code (EAK 2000).

For the following data: Seismic Risk Zone II, Soil class A, Importance Category S_2 = 1, Damping Ratio ζ = 5% and Foundation Factor θ = 1.0, calculate:

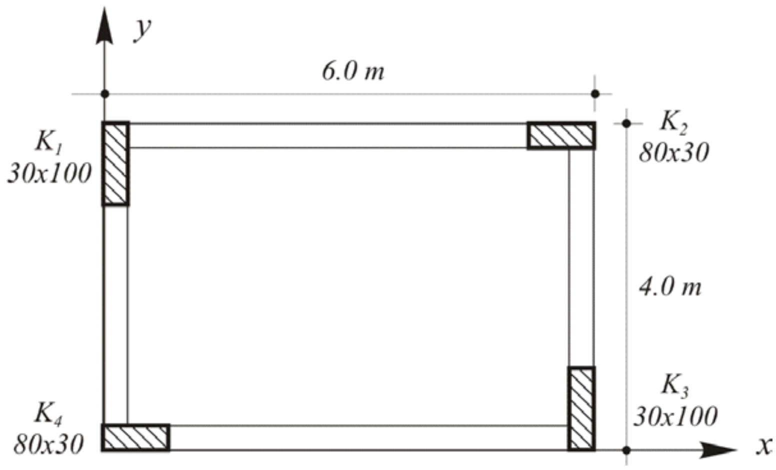
1. The design base shear force along with the design shear force and bending moment of column K_1.
2. The maximum expected displacement of the building.

B) After the construction, a recalculation of the seismic hazard showed that the max expected ground acceleration is 0.36g. In the case of having an earthquake event of this level, calculate:

1. The ductility developed by the structure.
2. The shear force and bending moment of column K_1.
3. The max displacement of structure during the earthquake.

Data

- The columns, of height h = 3.0 m, behave as double fixed elements.
- Young's modulus of elasticity $E = 30 \cdot 10^6$ kN/m^2, g = 10 m/sec^2.
- Direction of earthquake's design: x-x.
- Ignore the rotation of structure.
- The structure exhibits 40% overstrength.
- For mass calculation take also into account 30% of the live load.
- Permanent and live load: 10 kN/m^2 on the slab surface.
- Behavior factor (from EAK 2000) q = 3.5



Solution

A1) In general, the strategic procedure to be followed in cases of a seismic design, is to calculate the main parameters useful for the critical design values.

In our case, the mass, stiffness and natural period of the structure are the critical parameters before estimating the design base shear force.

Seismic Load Combination: **Q = g + 0.3·q**, where g the permanent given load and q is the live load. If B is the weight of the structure's slab, then

B = (6.0·4.0)m^2(10 kN/m^2 + 0.3·10 kN/m^2) = 312 kN. The mass of structure is

$$m = \frac{B}{g} = \frac{312\ kN}{10\ m/sec^2} = 31.2\ Mgr\ \ or\ \ t.$$

Since the seismic direction is x-x the stiffness parameters will be considered with respect to the y-y axis.

Second moments of inertia – Stiffness. It is:

$$J_{1y} = \frac{bh^3}{12} = \frac{1.0 \cdot 0.3^3}{12} = 2.25 \cdot 10^{-3}\ m^4$$

$$k_{1y} = \frac{12EJ_{1y}}{h^3} = \frac{12 \cdot 30 \cdot 10^6\ \left(\frac{kN}{m^2}\right) 2.25 \cdot 10^{-3}\ m^4}{3^3\ m^3} = 30000\ kN/m$$

Similarly

$$J_{2y} = \frac{bh^3}{12} = \frac{0.30 \cdot 0.8^3}{12} = 0.0128\ m^4$$

$$k_{2y} = \frac{12EJ_{2y}}{h^3} = \frac{12 \cdot 30 \cdot 10^6 \cdot 0.0128}{3^3} = 170667\ kN/m$$

Due to the point-symmetry of the structure's plan, $k_{1y} = k_{3y}$ and $k_{2y} = k_{4y}$. Therefore the total stiffness of structure is:

$K_{tot} = 2(k_{1y} + k_{2y}) = 2(30000 + 170667) = 401334$ kN/m.

The mass and stiffness parameters of a structure are enough to calculating its natural period T. Therefore:

$$T = 2\pi \sqrt{\frac{m}{k_{tot}}} = 2\pi \sqrt{\frac{31.2\ Mgr}{401334\ kN/m}} = 0.055\ sec.$$

Since $0 \leq T < T_1$ due to the soil class A of the structure, the design acceleration parameter $\Phi_d(T)/A\gamma_I$ is given by the equation:

$$\frac{\Phi_d(T)}{A \cdot \gamma_I} = 1 + \frac{T}{T_1}\left(\frac{\eta \cdot \theta \cdot \beta_0}{q} - 1\right)$$

where q = 3.5 for inelastic behavior of the structure. Substituting

$$\Phi_d(T) = 0.24g \cdot 1.0\left[1 + \frac{0.055}{0.10}\left(\frac{1.0 \cdot 1.0 \cdot 2.5}{3.5} - 1\right)\right] = 0.20g$$

The design base seismic horizontal force of the structure, is therefore

$$\mathbf{P_d} = m \cdot \Phi_d(T) = 31.2\ Mgr \cdot 0.20g = \mathbf{62.4\ kN}$$

The design shear force of column K_1 is

$$\boldsymbol{V_{d1}} = \frac{k_1}{k_{tot}} P_d = \frac{30000}{401334} 62.4 = \mathbf{4.66\ kN}$$

As a result, the design bending moment of the same column is obviously

$$\boldsymbol{M_{d1}} = V_{d1} \cdot \frac{h}{2} = 4.66 \frac{3}{2} = \mathbf{6.99\ kNm}$$

A2) The displacement of the structure corresponding to the yield point is

$$\delta_y = \frac{P_d}{k_{tot}} = \frac{62.4\ kN}{401334 \frac{kN}{m}} = 1.55 \cdot 10^{-4}\ m$$

Consequently the max displacement is:

$$\boldsymbol{\delta_{max}} = q \cdot \delta_y = 3.5 \cdot 1.55 \cdot 10^{-4} = \mathbf{5.43 \cdot 10^{-4}\ m}$$

B1) The ductility developed by the structure can be defined by the equation

$$\mu = q = \frac{P_{el}}{P_{real}}$$

where $\mathbf{P_{el}}$ is the elastic seismic horizontal force coming from the ideal elastic system, i.e. the new earthquake, presenting a max ground acceleration A' = 0.36g and a behavior factor q' = 1.0, while $\mathbf{P_{real}}$ is the real seismic horizontal force at first yield, coming from the old one at first yield, P_d, multiplied by the overstrength factor, which is **1.4**.

This factor (see p. 125 Penelis – Kappos) takes into account **the variability of the yield stress f_y and the probability of strain-hardening effects in the reinforcement**.

The earthquake acceleration of the **new seismic event**, derived for the same local conditions, is

$$\Phi'_{d(T)} = 0.36 \cdot 1.0 \left[1 + \frac{0.055}{0.10}\left(\frac{1.0 \cdot 1.0 \cdot 2.5}{1.0} - 1\right)\right] = 0.657g$$

$$P_{el} = m \cdot \Phi'_{d(T)} = 31.2 \cdot 0.657g = 204.98 \text{ kN}$$

$$P_{real} = 1.4 \cdot P_d = 1.4 \cdot 62.4 = 87.36 \text{ kN}$$

Therefore:

$$\boldsymbol{\mu'} = \frac{P_{el}}{P_{real}} = \frac{204.98}{87.36} = \mathbf{2.35}$$

B2) The real shear force and bending moment of the column K_1 can be similarly calculated

$$\boldsymbol{V_{real,1}} = \frac{k_1}{k_{tot}} P_{real} = \frac{30000}{401334} 87.36 = \mathbf{6.53\ kN}$$

$$\boldsymbol{M_{real,1}} = V_{real,1} \cdot \frac{h}{2} = 6.53 \frac{3}{2} = \mathbf{9.8\ kNm}$$

B3) Applying, as before, a similar way of thinking, the max displacement of the structure is

$$\delta'_{max} = \mu' \cdot \delta_{y,real}$$

where $\delta_{y,real}$ is the displacement of the structure corresponding to the first yield, which is

$$\delta_{y,real} = \frac{P_{real}}{k_{tot}} = \frac{87.36}{401334} = 2.18 \cdot 10^{-4}\ m$$

Consequently the maximum displacement is

$$\boldsymbol{\delta'_{max}} = \mu' \cdot \delta_{y,real} = 2.35 \cdot 2.18 \cdot 10^{-4} = \mathbf{5.12 \cdot 10^{-4}\ m}.$$

Exercise 7

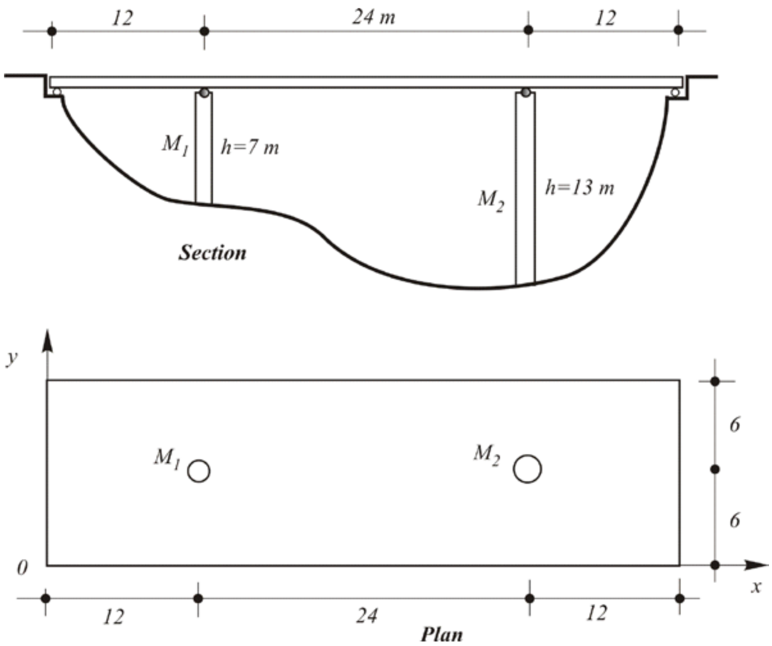
The R.C. bridge presented below, was designed according to Greek Seismic Code for zone I, soil class B, Importance category Σ3, Behavior factor q = 3 and Foundation factor θ = 1.0. During the design procedure the rotation of bridge was ignored.

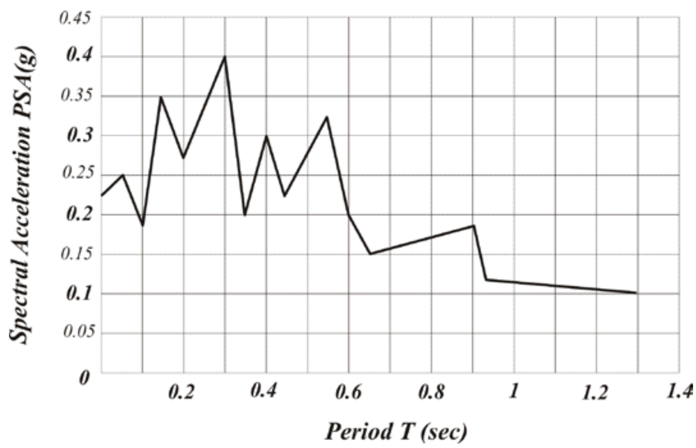
After the completion of the structure, an earthquake occurred in the area, the elastic response spectrum of which, for the y-y direction, is depicted in Fig. 2.

Calculate the **displacement ductility factor** for pier M_2 during the y-y seismic direction, taking also into account the rotation of the bridge.

Data and assumptions

- Uniformly distributed load on the bridge: 25 kN/m^2
- Young's modulus of elasticity: $E = 3 \cdot 10^7$ kN/m^2
- Overstrength factor for piers: 1.2
- Piers, presenting a circular cross section with diameter D = 1.7 m, behave as single fixed members (cantilevers)
- Ignore $K_{\omega i}$.





Solution

Seismic characteristics of structure **before earthquake**

Stiffness of piers along the y-y direction:

$$k_1 = \frac{3EJ_1}{h_1^3} = \frac{3 \cdot 3 \cdot 10^7 \cdot \pi \cdot 1.7^4/64}{7.0^3} = 107\,575.65\ kN/m$$

$$k_2 = \frac{3EJ_2}{h_2^3} = \frac{3 \cdot 3 \cdot 10^7 \cdot \pi \cdot 1.7^4/64}{13.0^3} = 16\,794.92\ kN/m$$

Therefore: $k_{tot} = k_1 + k_2 = 124\,370.57$ kN/m

Mass, period and seismic design acceleration of structure:

$$m = \frac{W}{g} = \frac{48 \cdot 12 \cdot 25}{10} = 1\,440\ Mgr$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1\,440}{124\,370.57}} = 0.676\ sec$$

For zone I, it is: A = 0.16g. Also given are:

Important factor: Σ3 = 1.15

Behavior factor: q = 3

Soil class A → θ = 1.0 and η = 1.0.

For soil class A, it is: T_1 = 0.10 sec, T_2 = 0.60 sec.

Since T=0.676 > T_2=0.60, we use equation 3 of the seismic code, i.e.

$$R_{d(T)} = \gamma_I A \frac{\eta\theta\beta_0}{q}\left(\frac{T_2}{T}\right)^{\frac{2}{3}} = 1.15 \cdot 0.16g\,\frac{1.0 \cdot 1.0 \cdot 2.5}{3}\left(\frac{0.60}{0.676}\right)^{\frac{2}{3}} = 0.142g$$

The horizontal seismic force on the y-y direction is therefore:

$F = m \cdot R_{d(T)} = 1\,440 \cdot 0.142g = 2\,044.8$ kN

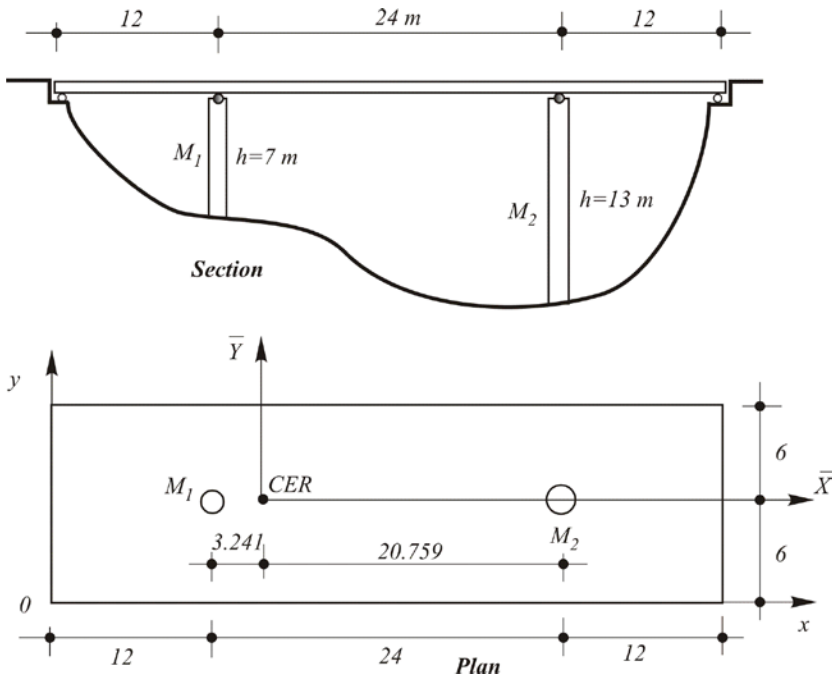
Consequently the **design shear force** developed at pier M_2, is:

$$V_d^{M_2} = F\frac{k_2}{k_{tot}} = 2\,044.8\ \frac{16\,794.92}{124\,370.57} = 276.13\ kN$$

Calculation of **shear force at pier M_2 after the seismic event**

Our target is to find out the relevant **displacement** of pier M_2, taking also into account the **rotation** of the bridge. For this reason we need to locate both the centre of gravity (CG) and the centre of elastic rotation (CER).

Initially we install a Cartesian coordinate system with its zero point on the bottom left of the deck.



Due to the symmetry of the structure's plan, both of the above centres will be on the horizontal axis of symmetry of the plan. Besides, the CG will be on the vertical axis of symmetry, presenting thus coordinates (24, 6) m.

In order to calculate the **abscissa** (horizontal distance from vertical axis) of CER, along with the components of the rotational stiffness, we fill up the following table:

Ki	**x_i (m)**	**Di_y (kN/m)**	**x_iDi_y (kN)**	$\boldsymbol{\bar{x} = x - x_{CER}}$ **(m)**	$\boldsymbol{\bar{x}^2 Di_y}$ **(kNm)**
M_1	12.0	107 575.65	1 290 907.8	-3.241	1 129 983.3
M_2	36.0	16 794.92	604 617.1	20.759	7 237 537
S U M		124 370.57	1 895 524.9		8 367 520.3

Before filling up the $\bar{x}$ (and possibly $\bar{y}$) field of the table we calculate the abscissa of CER, which is:

$$x_{CER} = \frac{\sum\left(x_i \cdot D_{i_y}\right)}{\sum D_{i_y}} = \frac{1\,895\,524.9}{124\,370.57} = 15.241\ m$$

Then, the coordinates of the ith column with respect to the CER system, are

$$\bar{x}_i = x_i - x_{CER}$$

(and $\bar{y}_i = y_i - y_{CER}$ respectively if we have **more** columns vertically).

Therefore we can proceed to filling up the last (two) column(s) of the table.

Similarly, the rotational stiffness of the bridge will be calculated through the form

$$k_\omega = \sum\left(D_{i_\omega} + \bar{x}_i^2 D_{i_y} + \bar{y}_i^2 D_x\right).$$

However, since the first term will be omitted, while, due to the symmetry, $\bar{y}_i$ = 0, it yields that

$$k_\omega = \sum\left(\bar{x}_i^2 D_{i_y}\right) = 8\,367\,520.3\ kNm/rad.$$

Now the displacement of pier M_2 due to the y-y earthquake, taking also into account the rotation of the structure, will be evaluated through the following formula, derived from page 52 of theory

$$v_{M_2} = \frac{P_y}{k_y} + \frac{-P_x \cdot \bar{y}_{CG} + P_y \cdot \bar{x}_{CG}}{k_\omega}\bar{x}_S$$

The **first** term comes from the **y-y shift** of the **<u>bridge</u>** as a whole, while the **second** expresses again the **y-y movement** of the **<u>Pier</u>** M_2 due to the bridge's rotation. It has to be noted that the second term is different from point to point, depending on the location of the pier with respect to the CER.

Making use of the given spectrum, for T = 0.676 sec, it yields that PSA = 0.155g.

The seismic elastic force, P_y, on the y-y direction is therefore:

$$P_y = m \cdot PSA = 1\,440 \cdot 0.155g = 2\,232\ kN$$

Besides, it is:

$\boldsymbol{\bar{x}_{CG}} = x_{CG} - x_{CER} = 24.0 - 15.241 = 8.759\, m,$

$\boldsymbol{\bar{y}_{CG}}$ = 0 while

$\boldsymbol{\bar{x}_S} = \bar{x}_{M_2} = x_{M_2} - x_{CER} = 36.0 - 15.241 = 20.759\ m.$

The **displacement** of pier M_2 is therefore:

$$v_{M_2} = \frac{P_y}{k_y} + \frac{P_y \cdot \bar{x}_{CG}}{k_\omega}\bar{x}_S = \frac{2\,232}{124\,370.57} + \frac{2\,232 \cdot 8.759}{8\,367\,520.3} 20.759 = 0.06645\, m$$

Consequently the relevant **elastic shear force** will be:

$$V_{M_2} = k_2 \cdot v_{M_2} = 16\,794.92 \cdot 0.06645 = 1\,116\, kN$$

The **displacement ductility factor** of pier M_2 can finally be estimated as:

$$\boldsymbol{\mu_{M_2}} = \frac{V_{M_2}}{1.2 \cdot V_d^{M_2}} = \frac{1\,116}{1.2 \cdot 276.13} = \mathbf{3.37}\,.$$

Exercise 8

The single storey framed structure illustrated in Fig. 1, was designed according to the Greek seismic code for zone II, soil class A and importance category 2. During the design procedure, the rotation of structure was ignored.

After the end of structure, an earthquake on the y-y direction occurred in the area, the elastic response spectrum of which is illustrated in Fig. 2.

Calculate the displacement ductility factor, developed at column K_4, for the above seismic direction y-y, taking also into account the rotation of the structure. The over-strength of the column was evaluated to be 20%.

Data: Weight of building 1000 kN, additional load at point A, 200 kN, Young's Modulus of Elasticity for reinforced concrete, $E = 30x10^6$ kN/m^2, g = 10 m/sec^2 and height of storey h = 3 m. The columns behave as double fixed elements.

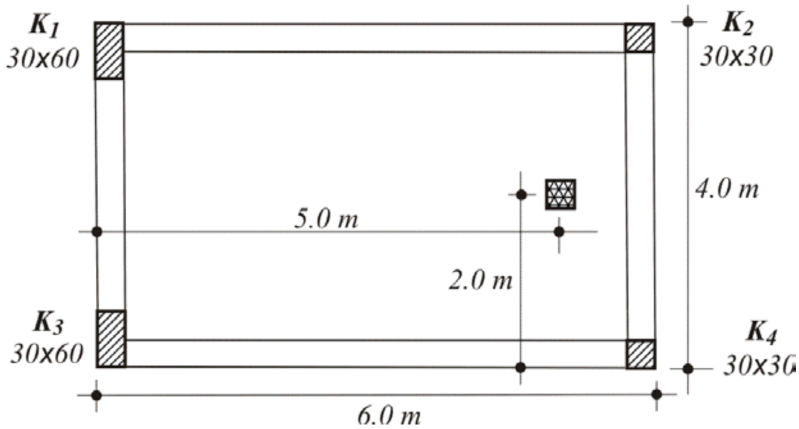


Figure 1

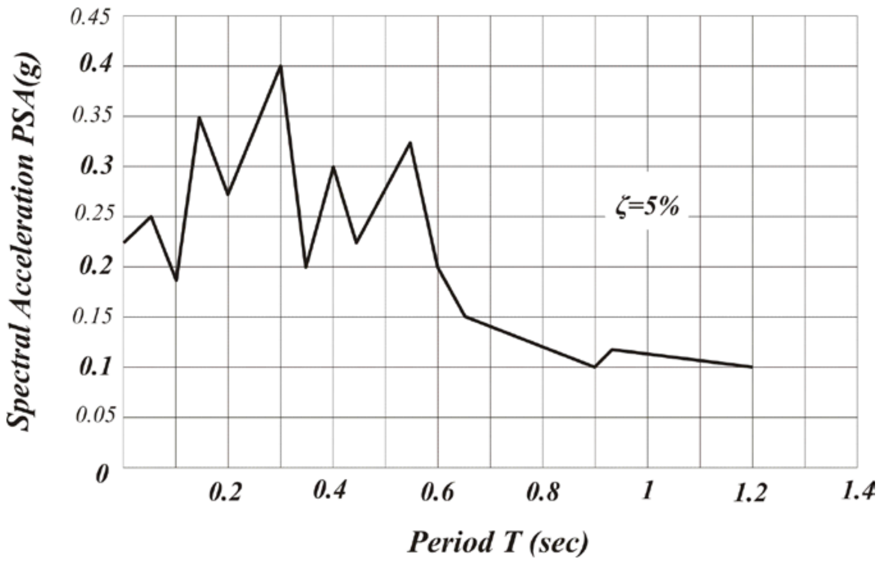


Figure 2

Solution

A) Evaluation of the design shear force for column K_4

Stiffness of columns:

$$K_{1,3}^{y} = \frac{12EI}{h^3} = \frac{12 \cdot 30 \cdot 10^6 \cdot 0.3 \cdot 0.6^3/12}{3.0^3} = 72000\ kN/m$$

$$K_{2,4}^{y} = \frac{12EI}{h^3} = \frac{12 \cdot 30 \cdot 10^6 \cdot 0.3^4/12}{3.0^3} = 9000\ kN/m$$

Total K = $2(K_{1,3}^{y} + K_{2,4}^{y})$ = 162000 kN/m

W = 1000 + 200 = 1200 kN

m = W/g = 1200/10 = 120 Mgr

$$T_y = 2\pi\sqrt{\frac{120}{162000}} = 0.171\ sec$$

For soil class A, it is T_1 = 0.10 sec and T_2 = 0.40 sec. Since $T_1 < T_y < T_2$,

It follows that $R_{d(Ty)} = \gamma_1 A \frac{\eta\theta\beta_0}{q}$, where:

Importance category, is Σ_2 ⇒ $\gamma_I = 1.0$

Zone II ⇒ A = 0.24g

Soil class A ⇒ θ = 1.0

q = 3.5 (framed structure)

η = 1.0 (reinforced concrete)

Therefore $R_{d(Ty)} = 1.0 \cdot 0.24 \cdot g \frac{1.0 \cdot 1.0 \cdot 2.5}{3.5} = 0.1714g$ and

$F = m \cdot R_{d(Ty)} = 120 \cdot 0.1714g = 205.68$ kN.

Consequently the design shear force for the column K_4 is:

$$V_{yd}^{K_4} = 205.68 \cdot \frac{9000}{162000} = 11.43\ kN.$$

B) Evaluation of V_y^{K4} after the earthquake

For calculating the displacement ductility factor of column K_4 we need to estimate the relevant **elastic shear force** of the column, which will be derived from its **total displacement**.

Using the elastic spectrum (Fig.2), for Ty = 0.171 sec ⇒ PSA = 0.30g.

Now we have to take into account the rotation of the structure. For this reason we install a **Cartesian** system with its point of origin (0,0) at the bottom left end of the column K_3.

Coordinates of the centre of gravity (centroid) K:

The structure presents symmetry of the columns' loads F_i with respect to horizontal axis. Hence the coordinate of the centroid is:

$$\mathbf{y_K = 2.0\ m.}$$

Now, if S_y is the first moment of area (weights) of the structure with respect to the y-axis, then the abscissa (horizontal distance from axis) of the centroid is:

x_K = $S_y/\Sigma F_i = \Sigma(F_i \cdot x_i)/\Sigma F_i = (1000 \cdot 3.0 + 200 \cdot 5.0)/1200$ = **3.33 m**

Coordinates of the center of elastic rotation (CER), E:

The structure also presents a stiffness-symmetry of columns with respect to horizontal axis.

Therefore the coordinate (vertical distance from horizontal axis) of its CER, is:

y_E = 2.0 m.

The following table comprises the procedure to be followed in order to calculate the **abscissa** of CER and then the **displacement** of **K_4**, where:

- x_i, y_i are the coordinates of the i[th] column's cross sectional cendroid with respect to the Cartesian system,
- Di_x, Di_y are the stiffnesses of the i[th] column with respect to the x and y direction of the Cartesian system respectively,
- $\bar{x}$, $\bar{y}$ are the coordinates of the columns cendroid with respect to the $\bar{X}$ and $\bar{Y}$ axes (that have as origin the CER) parallel to x and y.

Ki	x_i (m)	y_i (m)	Di_x (kN/m)	Di_y (kN/m)	$x_i Di_y$	$\bar{x}_i$ (m)	$\bar{y}_i$ (m)	$\bar{x}_i^2 Di_y$ (kNm)	$\bar{y}_i^2 Di_x$ (kNm)
K_1	0.15	3.70	18000	72000	10800	-0.63	1.70	28576.8	52020
K_2	5.85	3.85	9000	9000	52650	5.07	1.85	231344.1	30802.5
K_3	0.15	0.30	18000	72000	10800	-0.63	-1.70	28576.8	52020
K_4	5.85	0.15	9000	9000	52650	5.07	-1.85	231344.1	30802.5
S U M			54000	162000	126900			519841.8	165645

Before we fill up the $\bar{x}$ and $\bar{y}$ fields of the table, we calculate the abscissa of CER, which is:

$$x_E = \frac{\sum(x_i \cdot D_{i_y})}{\sum D_{i_y}} = \frac{126900}{162000} = 0.78m$$

Then, the coordinates of the i[th] column with respect to the CER system, are

$\bar{x}_i = x_i - x_E$ and $\bar{y}_i = y_i - y_E$ respectively.

Therefore we proceed to filling up the last two columns of the table.

If K_ω is the rotational stiffness of the structure, then:

$$K_\omega = \sum \left(D_{i_\omega} + \bar{x}_i^2 D_{i_y} + \bar{y}_i^2 D_{i_x} \right) = 519841.8 + 165645 = 685486.8\ kNm$$

(where the first term, being too small compared to the others, has been omitted).

The displacement of column K_4 on the y-y direction, taking into account both the shift (due to seismic force) and the rotation of the structure, is

$$u_y^{K_4} = \frac{P_y}{K_y} + \frac{P_y}{K_\omega} \bar{x}_K \cdot \bar{x}_{K_4}$$

The first term comes from the vertical movement of slab as a whole, while the second expresses the vertical movement of the column K_4 due to the slab's rotation. It has to be noted that the second term is different from point to point, depending on the location of the column with respect to the CER.

The seismic elastic force along the y-y axis is:

$$P_y = m \cdot PSA = 120 \cdot 0.30g = 360 \text{ kN}.$$

Besides, $K_y = \Sigma Di_y$ and

$\bar{x}_K = x_K - x_E$ = 3.33 – 0.78 = 2.55 m. Therefore:

$$u_y^{K_4} = \frac{360}{162000} + \frac{360}{685486.8} 2.55 \cdot 5.07 = 0.009\ m = 0.9\ cm$$

The relevant elastic shear force, P_{el}, for the column K_4 is therefore

$$P_{el}^{K_4} = D_y \cdot u_y = 9000 \cdot 0.009 = 81 \text{ kN}.$$

Finally, the corresponding ductility factor for column K_4 is

$$\boldsymbol{q_y^{K_4}} = \frac{P_{el}^{K_4}}{1.2 V_{yd}^{K_4}} = \frac{81}{1.2 \cdot 11.4} = \mathbf{5.9}\,.$$

Exercise 9

The frame illustrated below consists of weightless columns of a common square section. The columns, single fixed at A and double fixed at D, support a stiff girder.

The system, being designed against earthquake, gave the following acceleration spectrum:

$$S_a/g = \begin{cases} 0.2 + 4\,T & \text{for} \quad 0 \le T \le 0.2 \text{ sec} \\ 1.0 & \text{for} \quad 0.2 \le T \le 0.60 \text{ sec} \\ 0.60\,/\,T & \text{for} \quad T \ge 0.60 \text{ sec.} \end{cases}$$

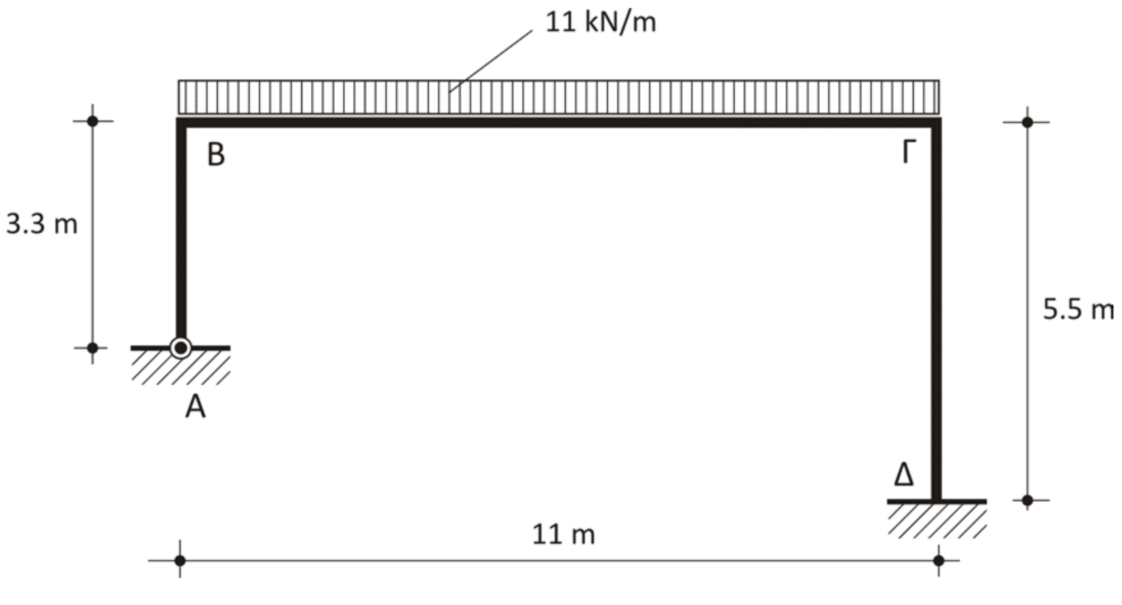
(**a**) Build up the corresponding Displacement design spectrum (in cm).

(**b**) Determine the minimum cross sectional side of columns so that the maximum displacement, (u_{max}), is not greater than 4 cm.

(**c**) Calculate the maximum bending moment developed to each column due to the seismic excitation.

Data:

- $E = 10^7$ kN/m^2 ,
- $g = 10$ m/sec^2.

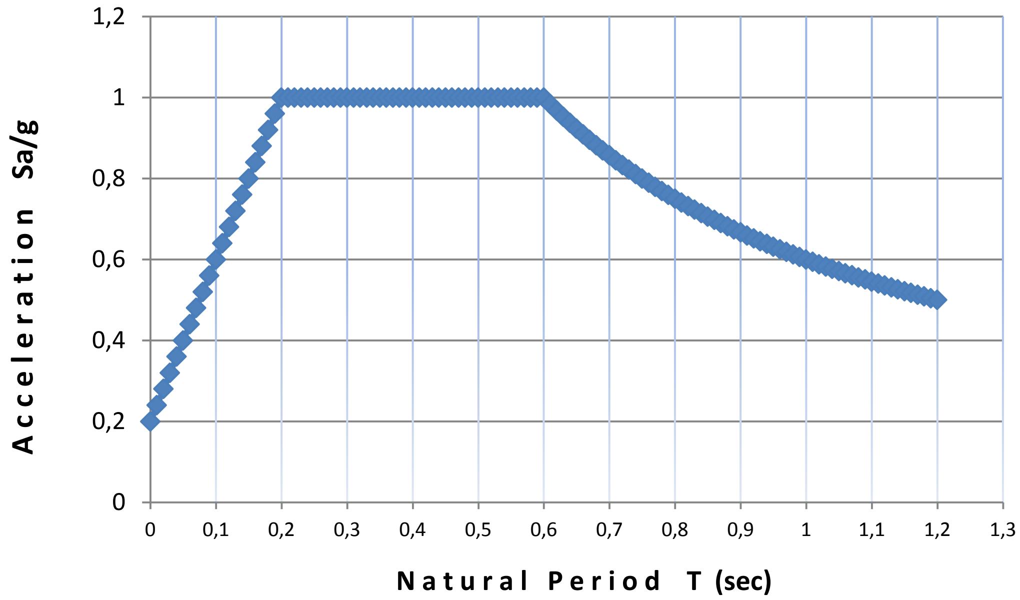


<u>**Solution**</u>

The design acceleration spectrum,

$$
S_a/g = \begin{cases} 0.2 + 4\,T & \text{for} \quad 0 \le T \le 0.2 \text{ sec} \\ 1.0 & \text{for} \quad 0.2 \le T \le 0.60 \text{ sec} \\ 0.60\,/\,T & \text{for} \quad T \ge 0.60 \text{ sec,} \end{cases}
$$

after a data process through or without EXCEL, leads to the following graph:



(a) Using the pseudo-spectral relation

$$S_a = \omega^2 S_d = \frac{4\pi^2}{T^2} S_d$$

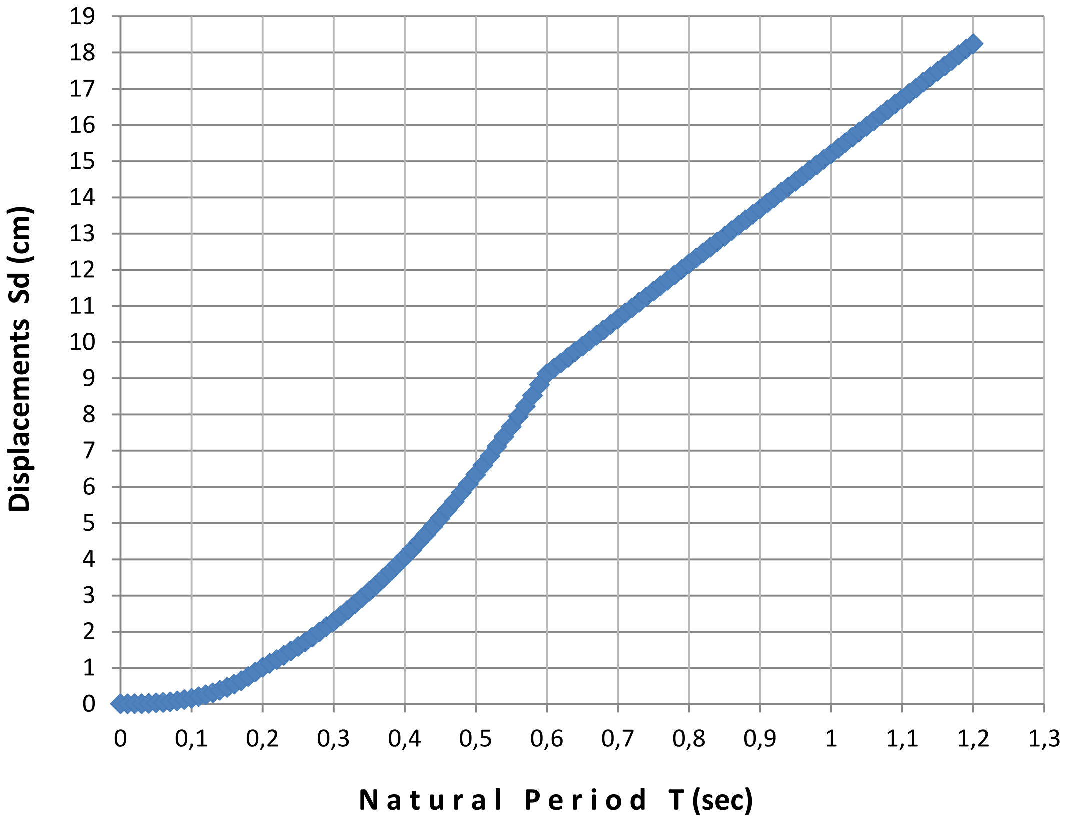
and solving for S_d, yields the displacement relation

$$S_d = S_a \cdot \frac{T^2}{4\pi^2}$$

which provides the values of displacements from the corresponding values of natural periods. Therefore the **displacement** spectrum takes the form

$$
S_d = \begin{cases} \dfrac{g(0.2+4T)T^2}{4\pi^2} & \text{for} \quad 0 \le T \le 0.20 \text{ sec} \\[2ex] \dfrac{g \cdot T^2}{4\pi^2} & \text{for} \quad 0.20 \le T \le 0.60 \text{ sec} \\[2ex] 0.60 \cdot T \cdot g\,/\,4\pi^2 & \text{for} \quad T \ge 0.60 \text{ sec,} \end{cases}
$$

which, after a similar process through EXCEL, leads to the following graph:



(b) The demand for a maximum displacement of **4 cm**, corresponds, as yielded from the above graph, obviously to the **second** branch of the spectrum, which starts from the value Sd = 1 cm and goes on upwards until 9 cm. The displacement limits of this branch can be calculated by substituting the limit values of T on the corresponding equation.

Therefore, for the above limited value of 4 cm, it holds that:

$$0.04 = \frac{g \cdot T^2}{4\pi^2}$$

wherefrom it yields

$$T = \sqrt{\frac{4\pi^2 \cdot 0.04}{10}} = 0.397\, sec,$$

a value, which is verified from the above displacement spectrum. Therefore:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.397} = 15.83\, rad/sec.$$

The oscillating mass is:

$$m = q \cdot \frac{L}{g} = 11 \cdot \frac{11}{10} = 12.1\, tn.$$

The total stiffness of columns, responding to the maximum displacement of 4 cm, is:

$$\omega = \sqrt{\frac{k_s}{m}} \quad \rightarrow \quad k_s = m \cdot \omega^2 = 12.1 \cdot 15.83^2 = 3032.13\, kN/m.$$

Obviously k_s is the sum of stiffnesses that yield respectively from the single fixed column k_1 and the double fixed k_2, each one of which is:

$$k_1 = \frac{3EI}{h_1^3} = \frac{3 \cdot 10^7 \cdot I}{3.3^3} = 834\,794.22 \cdot I$$

$$k_2 = \frac{12EI}{h_2^3} = \frac{12 \cdot 10^7 \cdot I}{5.5^3} = 721\,262.21 \cdot I$$

Therefore $k_s = k_1 + k_2 = (834\,794.22 + 721\,262.21) \cdot I$. →

$$3032.13 = 1\,556\,056.43 \cdot \boldsymbol{I} \quad \rightarrow \quad \mathbf{I} = 0.0019486\ \mathrm{m}^4.$$

Due to the square cross section (a·a) of columns, the second moment of area with respect to the cendroidal axis is

$$I = \frac{a^4}{12} \quad \rightarrow \quad \boldsymbol{a} = \sqrt[4]{12 \cdot 0.0019486} = \mathbf{0.391\,m}.$$

(c) Having calculated the cross sectional side, the stiffness for each one of the columns is:

$$k_1 = 834\,794.22 \cdot I = 834\,794.22 \cdot 0.0019486 = 1\,626.68\ kN/m$$

$$k_2 = 721\,262.21 \cdot I = 721\,262.21 \cdot 0.0019486 = 1\,405.45\, kN/m.$$

Therefore the corresponding maximum shear forces and bending moments of the columns are:

Single fixed:

$$V_1 = k_1 \cdot S_d = 1\,626.68 \cdot 0.04 = 65.07\ \mathrm{kN}$$

$$\mathbf{M_1} = V_1 \cdot h = 65.07 \cdot 3.3 = \mathbf{214.73\ kNm}$$

Double fixed:

$$V_2 = k_2 \cdot S_d = 1\,405.45 \cdot 0.04 = 56.22\ \mathrm{kN}$$

$$\mathbf{M_2} = V_1 \cdot h/2 = 56.22 \cdot 5.5/2 = \mathbf{154.61\ kNm}$$

Exercise 10

The three-storey R.C. building of Fig.1 was designed according to the Greek Seismic Code, for the following parameters:

- Seismic zone: II,
- Soil class: B,
- Importance factor $\gamma_I = 1$,
- Foundation factor: $\theta = 1$ and
- Damping ratio: $\zeta = 5\%$.

On the roof of the building a small R.C. floor is going to be constructed, the plan of which is depicted in Fig. 2.

For a seismic direction **y-y**, calculate the bending moments for each column of the roof structure. The spectrum shown in Fig. 3 is referred to the roof of the 3-storey building.

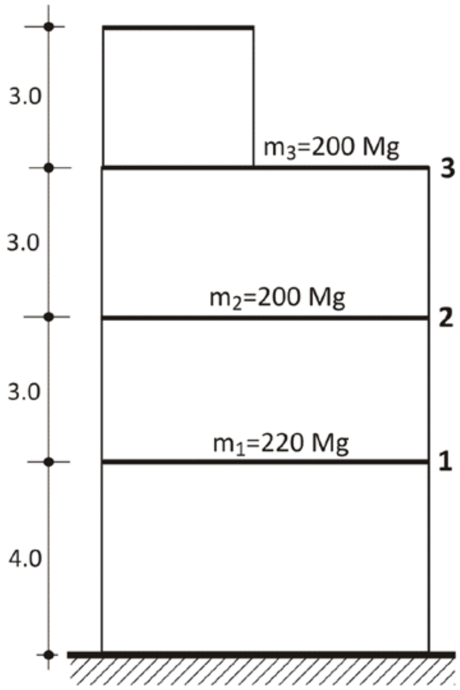


Fig. 1 Vertical section of the building

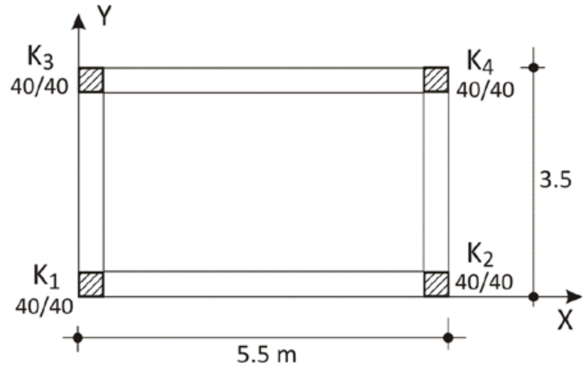


Fig. 2 Plan of the roof structure

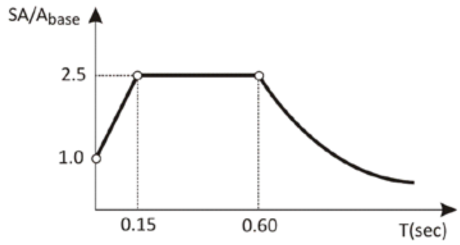


Fig. 3 Spectrum on the roof of the three-storey building

Data and assumptions

- The roof structure, behaving as SDOF system, does not affect the overal status of the existing building.
- Natural period of the three-storey building: T=0.26 sec.
- Total uniform load on the slab of the roof structure: 11 kN/m^2.
- Columns behave as double fixed elements.
- Young's modulus of elasticity for R.C. $E=27\cdot10^6 kN/m^2$.
- $g = 10 m/sec^2$.

Solution

A) Seismic characteristics of the **existing building**

Total mass: 2·200 + 1·220 = 620 Mgr

Natural period: T_b = 0.26 sec

Zone: II → A = 0.24g

Importance factor: γ_I = 1.0

Behavior factor: q = 3.5 (frame structure) → ζ = 5% → η = 1.0

Foundation factor: θ = 1.0.

Soil class: B → T_1 = 0.15 sec, T_2 = 0.60 sec. Since 0.15 < T_b < 0.60 we use eq. 2

The design acceleration is therefore:

$$R_{d(T_b)} = \gamma_I \cdot A \frac{\eta \cdot \theta \cdot \beta_0}{q} = 1.0 \cdot 0.24g \frac{1.0 \cdot 1.0 \cdot 2.5}{3.5} = 0.171g$$

Consequently the design base seismic horizontal force is:

$$F = m \cdot R_{d(Tb)} = 620 \cdot 0.171g = 1060.2 \text{ kN} = V_0.$$

The above shear base force, V_0, is, according to the equivalent static method, distributed to each floor through the formula

$$F_i = V_0 \frac{m_i \cdot z_i}{\sum_{j=1}^{n} m_j \cdot z_j}$$

$$F_1 = 1060.2 \frac{220 \cdot 4}{220 \cdot 4 + 200 \cdot 7 + 200 \cdot 10} = \frac{1060.2}{4280} 880 = 217.99\ kN$$

$$F_2 = 1060.2 \frac{200 \cdot 7}{220 \cdot 4 + 200 \cdot 7 + 200 \cdot 10} = \frac{1060.2}{4280} 1400 = 346.79\ kN$$

$$F_3 = 1060.2 \frac{200 \cdot 10}{220 \cdot 4 + 200 \cdot 7 + 200 \cdot 10} = \frac{1060.2}{4280} 2000 = 495.42\ kN$$

Check: ΣF_i = 1060.2 kN

The seismic force of the third floor, F_3, develops obviously an acceleration, α_3, on this level, which is:

$$\boldsymbol{a_3} = \frac{F_3}{m_3} = \frac{495.42\ kN}{200\ Mgr} = 2.477 \frac{m}{sec^2} = \mathbf{0.2477g}$$

B) Seismic characteristics of the **roof structure**

Stiffness: $k = 4k_1$, i.e.

$$k = 4\left(\frac{12 \cdot 27 \cdot 10^6 \cdot 0.4^4/12}{3.0^3}\right) = 4 \cdot 25\,600 = 102\,400\ kN/m$$

Mass: m = 5.5·3.5·11/10 = 21.175 Mgr

Period: $T = 2\pi\sqrt{\frac{m}{k}}$ = $2\pi\sqrt{\frac{21.175}{102400}} = 0.09\ sec$

From the spectrum referred to the roof of the building, for T = 0.09 sec, through interpolation, it yields

$SA/A_{base} = 1.9$. Consequently

$SA = 1.9 \cdot A_{base} = 1.9 \cdot 0.2477g = 0.47g$.

Therefore the total seismic force, P, of the roof structure, is:

$$P = \frac{m \cdot PSA}{q} = \frac{21.175\ Mgr \cdot 0.47 \cdot 10\ m/sec^2}{3.5} = 28.44\ kN$$

The value of q has been taken equal to **3.5** to comply with the rest of the structure.

This force develops a shear force to each column, which is:

$$V_{1,2,3,4} = P\frac{1}{4} = 28.44\frac{1}{4} = 7.11\ kN$$

Consequently, the corresponding bending moments developed to each column is:

$$\boldsymbol{M_{1,2,3,4}} = V_{1,2,3,4}\frac{h}{2} = 7.11\frac{3}{2} = \mathbf{10.67\ kNm}\,.$$

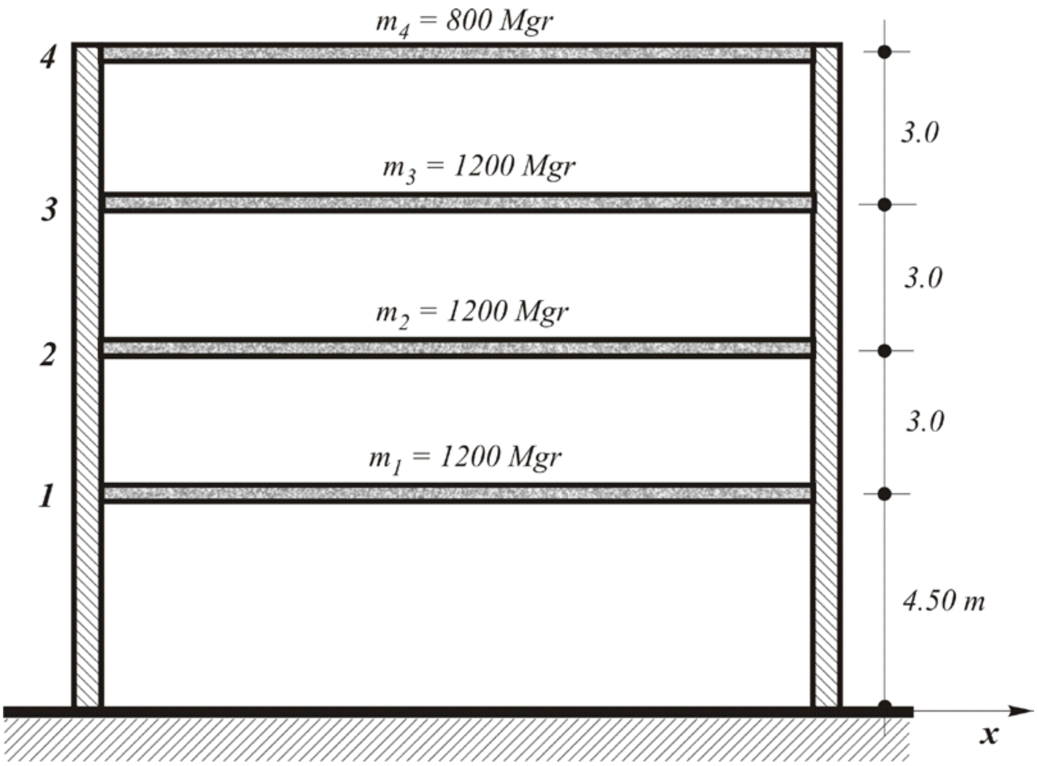
Exercise 11

The four-storey building illustrated below is a R.C. structure.

1. Calculate the total shear base force along with the total shear forces and bending moments acting on each pair of columns, through the Equivalent Static Method.
2. Construct the corresponding diagrams of shear forces and bending moments.

Data:

- Natural period $T = 0.65$ sec,
- Seismic Zone I,
- Soil Class B,
- Importance category S2,
- Damping Ratio $\zeta = 5\%$,
- Foundation factor $\theta = 1.0$ and
- $g = 10$ m/sec^2.



Solution

Equivalent Static Method

Using the given data, we apply the following parameters:

- Seismic Risk Zone I: $\Rightarrow$ Ground Seismic Acceleration: A = 0.16g
- Soil Class B: $\Rightarrow$ Characteristic Periods T_1 = 0.15 sec and T_2 = 0.60 sec
- Importance Category S_2 $\Rightarrow$ Importance Factor γ_I = 1.0
- Damping ratio: ζ = 5% and
- Foundation Factor: θ = 1.0

For natural period T = 0.65 sec > T_2, the design spectrum acceleration parameter, taken from equation (2.1.c) (EAK 2000), is:

$$\Phi_{d(T)} = \gamma_I \cdot A \frac{\eta \cdot \theta \cdot \beta_0}{q} \left(\frac{T_2}{T}\right)^{\frac{2}{3}} = 1.0 \cdot 0.16g \frac{1.0 \cdot 1.0 \cdot 2.5}{3.5} \left(\frac{0.60}{0.65}\right)^{\frac{2}{3}} = 1.08 \frac{m}{sec^2}$$

The total mass of the structure is: m_{tot} = 1200·3 + 800 = 4400 Mgr.

Therefore the structure's shear base seismic force is:

P = $m_{tot} \cdot \Phi_{d(T)}$ = 1.08·4400 = **4752 kN**

According to theory, due to the fact that T < 1 sec, the above shear base force is distributed along the height of the structure according to the formula:

$$F_i = P \frac{m_i \cdot z_i}{\sum(m_i \cdot z_i)}$$

where m_i is the mass of the i^{th} storey and z_i is its corresponding height from base of the structure. Here it is:

$$\sum(m_i \cdot z_i) = 1200 \cdot 4.5 + 1200 \cdot 7.5 + 1200 \cdot 10.5 + 800 \cdot 13.5 = 37800$$

The seismic horizontal force for each storey is therefore:

$$F_1 = P \frac{m_1 \cdot z_1}{\sum(m_i \cdot z_i)} = 4752 \frac{1200 \cdot 4.5}{37800} = 678.86\ kN$$

$$F_2 = P \frac{m_2 \cdot z_2}{\sum(m_i \cdot z_i)} = 4752 \frac{1200 \cdot 7.5}{37800} = 1131.43\ kN$$

$$F_3 = P \frac{m_3 \cdot z_3}{\sum(m_i \cdot z_i)} = 4752 \frac{1200 \cdot 10.5}{37800} = 1584\ kN$$

$$F_4 = P \frac{m_4 \cdot z_4}{\sum(m_i \cdot z_i)} = 4752 \frac{800 \cdot 13.5}{37800} = 1357.71\ kN$$

which, for checking, gives the sum of 4752 kN.

The corresponding allocation of bending moments for each storey comes as a result of the above shear forces.

Since $M_{i,foot} = M_{i,head} = M_i = F_{i+} \cdot h_i/2$, where F_{i+} is the sum of the i[th] plus all the above it seismic horizontal forces, it is:

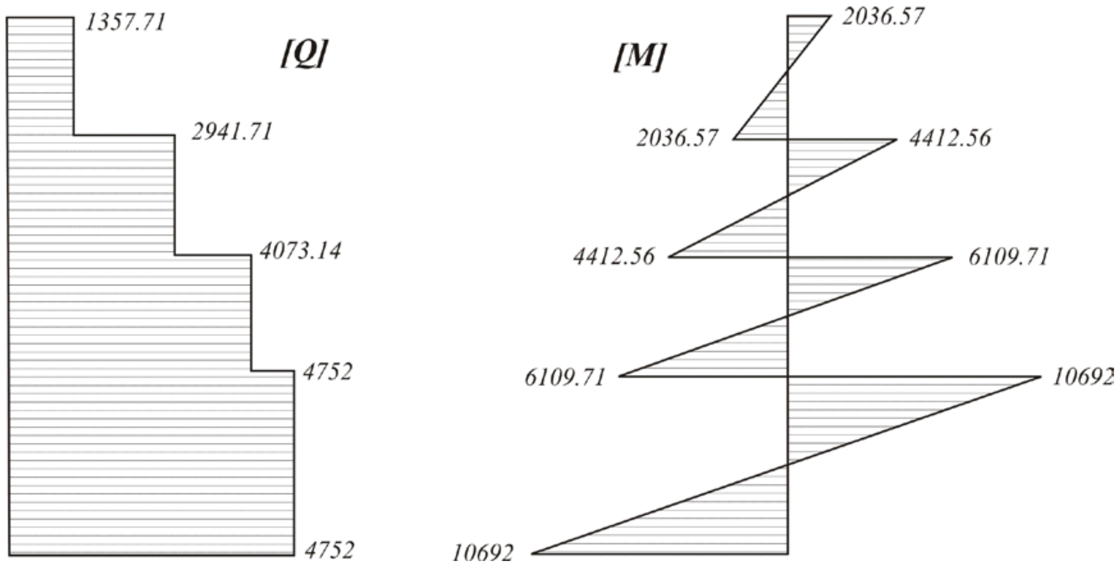
$$M_4 = F_4 \frac{h_4}{2} = 1357.71 \frac{3}{2} = 2036.57\ kNm$$

$$M_3 = (F_3 + F_4) \frac{h_3}{2} = (1357.71 + 1584) \frac{3}{2} = 4412.56\ kNm$$

$$M_2 = (F_2 + F_3 + F_4) \frac{h_2}{2} = (1131.43 + 1357.71 + 1584) \frac{3}{2} = 6109.71\ kNm$$

$$M_1 = (F_1 + F_2 + F_3 + F_4) \frac{h_1}{2} = (4752) \frac{4.5}{2} = 10692\ kNm$$

Following are the corresponding shear force and bending moment diagrams.



Exercise 12

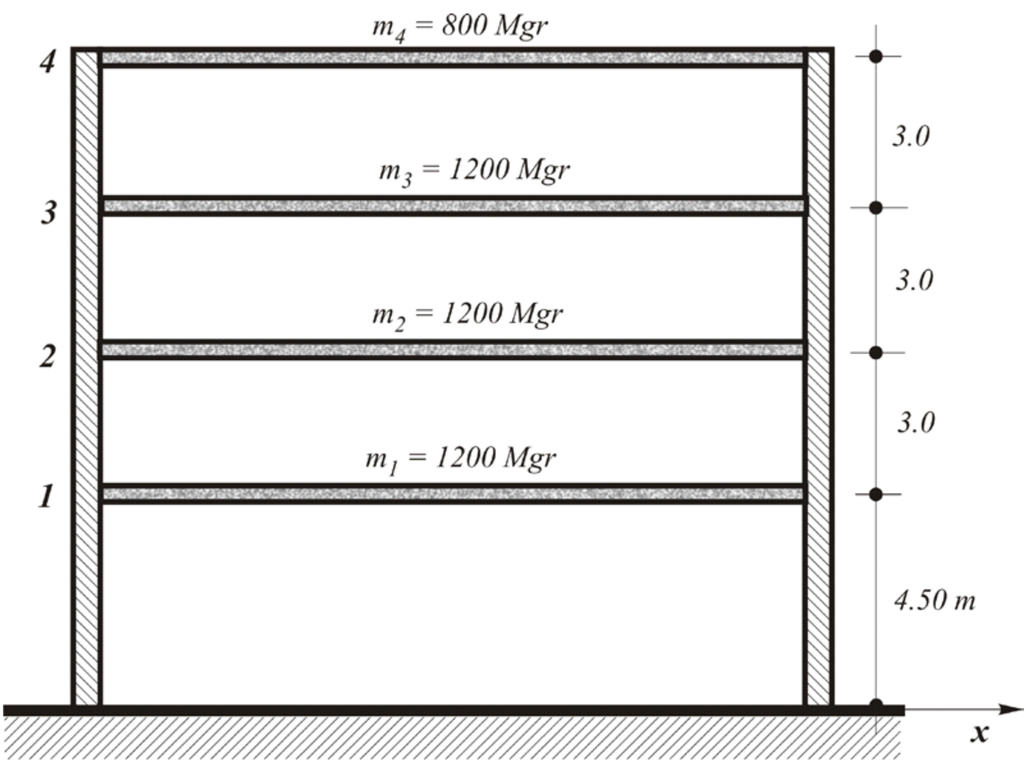
The four-storey building illustrated below is a R.C. structure.

1. Examine if the dynamic (modal superposition) method is applicable, using only the first two modal shapes and then calculate the total shear base force along with the total shear forces and bending moments acting on each pair of columns.
2. Construct the corresponding shear force and bending moment diagrams of columns.
3. Compare your results with those of previous exercise and comment accordingly.

Data:

- Seismic Zone I, Soil Class B, Importance category S2, Damping Ratio ζ = 5%, foundation factor θ = 1.0 and g = 10 m/sec^2.
- Natural periods of the first two modal shapes: T1 = 0.65 sec and T2 = 0.17 sec respectively.
- Eigenvalues of the first two modal shapes:

$$\{\Phi_1\} = \begin{Bmatrix} \varphi_{41} \\ \varphi_{31} \\ \varphi_{21} \\ \varphi_{11} \end{Bmatrix} = \begin{Bmatrix} 1.00 \\ 0.88 \\ 0.62 \\ 0.36 \end{Bmatrix} \qquad \{\Phi_2\} = \begin{Bmatrix} \varphi_{42} \\ \varphi_{32} \\ \varphi_{22} \\ \varphi_{12} \end{Bmatrix} = \begin{Bmatrix} 1.00 \\ 0.32 \\ -0.42 \\ -0.86 \end{Bmatrix}$$



Solution

The total mass of the structure is: m_{tot} = 3·1200 + 800 = 4400 Mgr.

1. Design of seismic values

Generalized masses

For the two given modal shapes, each generalized mass M_i, (i = 1, 2), playing the role of a "mass" at the i^{th} natural oscillation of the system, is:

$$M_1 = m_1\varphi_{11}^2 + m_2\varphi_{21}^2 + m_3\varphi_{31}^2 + m_4\varphi_{41}^2$$
$$= 1200 \cdot 0.36^2 + 1200 \cdot 0.62^2 + 1200 \cdot 0.88^2 + 800 \cdot 1.0^2$$
$$= 2346.08\ Mgr$$

$$M_2 = m_1\varphi_{12}^2 + m_2\varphi_{22}^2 + m_3\varphi_{32}^2 + m_4\varphi_{42}^2$$
$$= 1200 \cdot (-0.86)^2 + 1200 \cdot (-0.42)^2 + 1200 \cdot 0.32^2 + 800 \cdot 1.0^2$$
$$= 2022.08\ Mgr$$

Excitation factors

These are intermediate modal magnitudes, helping to calculate the horizontal forces for each level; their values are:

$$L_1 = m_1\varphi_{11} + m_2\varphi_{21} + m_3\varphi_{31} + m_4\varphi_{41}$$
$$= 1200 \cdot 0.36 + 1200 \cdot 0.62 + 1200 \cdot 0.88 + 800 \cdot 1.0 = 3032$$

$$L_2 = m_1\varphi_{12} + m_2\varphi_{22} + m_3\varphi_{32} + m_4\varphi_{42}$$
$$= -1200 \cdot 0.86 - 1200 \cdot 0.42 + 1200 \cdot 0.32 + 800 \cdot 1.0 = -352$$

Participation factors

The participation factors, v_i, are largely decreased by an increase of the modular number, i. In general, their value is $v_i = L_i/M_i$, i.e:

$$v_1 = \frac{L_1}{M_1} = \frac{3032}{2346.08} = 1.292$$

$$v_2 = \frac{L_2}{M_2} = \frac{-352}{2022.08} = -0.174$$

Check: $\mathbf{v_1 + v_2}$ = 1.112 **≈ 1.0**

Active Modal masses

The active modal mass, M_{ai}, is, for each modal shape, a quantitative criterion of the maximum energy of deformation and constitutes an index of its significance.

In practice it yields the number of significant modal shapes to be taken into account, ignoring all the others. The sum of all the active modal masses has a constant value, Ms, close to the sum of the real masses. In general, the value of the i^{th} modal mass, M_{ai}, is $M_{ai} = v_i^2 \cdot M_i = L_i^2/M_i$, i.e:

$$M_{a1} = \frac{L_1^2}{M_1} = \frac{3032^2}{2346.08} = 3918.5\,Mgr$$

$$M_{a2} = \frac{L_2^2}{M_2} = \frac{(-352)^2}{2022.08} = 61.28\,Mgr$$

Check: $\mathbf{Ms = M_{a1} + M_{a2}}$ = 3918.5 + 61.28 = 3979.78 ≈ $\mathbf{4400 = m_{tot}}$

It is: $M_{\alpha 1} + M_{\alpha 2}$ = 3979.78 Mgr and

$0.9 \cdot m_{tot} = 0.9 \cdot 4400 = 3960$ Mgr

Since: $M_{a1} < 0.9 \cdot m_{tot}$, but $M_{a1} + M_{a2} > 0.9 \cdot m_{tot}$,

it follows that the first modal shape is not enough, while **the first two** modal shapes are adequate to calculating the seismic response, using the given data:

- Zone I factor = 0.16 ⇒ A = 0.16g
- Soil class B ⇒ T_1 = 0.15 sec, T_2 = 0.60 sec
- Importance Category S_2 ⇒ γ_I = 1.0
- Frame structure ⇒ q = 3.5

Solution for the 1st modal shape

The natural period for this mode is $\mathbf{T_1 = 0.65}$ sec. Since $T_1 > 0.60$, it follows that the maximum design acceleration for the first mode is:

$$R_{d(T_1)} = \gamma_I \cdot A \frac{\eta \cdot \theta \cdot \beta_0}{q} \left(\frac{T_2}{T_1'}\right)^{\frac{2}{3}} = 1.0 \cdot 0.16g\,\frac{2.50}{3.5}\left(\frac{0.60}{0.65}\right)^{\frac{2}{3}} = 0.108g$$

Following the procedure presented on page 69 of handouts, the corresponding seismic forces per floor due to the 1st mode are:

$$P_{1,1} = m_1 \cdot \varphi_{1,1} \frac{L_1}{M_1} S_{a1} = \mathbf{1200 \cdot 0.36} \cdot 1.292 \cdot 0.108 \cdot 10 = 602.8\,kN$$

$$P_{2,1} = m_2 \cdot \varphi_{2,1} \frac{L_1}{M_1} S_{a1} = \mathbf{1200 \cdot 0.62} \cdot 1.292 \cdot 0.108 \cdot 10 = 1038.15\,kN$$

$$P_{3,1} = m_3 \cdot \varphi_{3,1} \frac{L_1}{M_1} S_{a1} = \mathbf{1200 \cdot 0.88} \cdot 1.292 \cdot 0.108 \cdot 10 = 1473.5\,kN$$

$$P_{4,1} = m_4 \cdot \varphi_{4,1} \frac{L_1}{M_1} S_{a1} = \mathbf{800 \cdot 1.0} \cdot 1.292 \cdot 0.108 \cdot 10 = 1116.29\,kN$$

The first modal shape contribution to the shear base seismic force is thus:

$$V_{01} = \sum F_{i1} = 4230.74\,kN$$

Solution for the 2nd modal shape

Similarly, the natural period for this mode is **T_2 = 0.17** sec. Since 0.15 < T_2 < 0.60, it follows:

$$R_{d(T_2)} = \gamma_I \cdot A \frac{\eta \cdot \theta \cdot \beta_0}{q} = 1.0 \cdot 0.16g \frac{1.0 \cdot 1.0 \cdot 2.5}{3.5} = 0.114g$$

In the same way, following the procedure presented on page 69 (handouts), the corresponding seismic forces per floor due to the 2nd mode are:

$$P_{1,2} = m_1 \cdot \varphi_{1,2} \frac{L_2}{M_2} S_{a2} = \mathbf{1200 \cdot (-0.86)} \cdot (-0.174) \cdot 0.114 \cdot 10 = 204.71\ kN$$

$$P_{2,2} = m_2 \cdot \varphi_{2,2} \frac{L_2}{M_2} S_{a2} = \mathbf{1200 \cdot (-0.42)} \cdot (-0.174) \cdot 0.114 \cdot 10 = 99.97\ kN$$

$$P_{3,2} = m_3 \cdot \varphi_{3,2} \frac{L_2}{M_2} S_{a2} = \mathbf{1200 \cdot 0.32} \cdot (-0.174) \cdot 0.114 \cdot 10 = -76.17\ kN$$

$$P_{4,2} = m_4 \cdot \varphi_{4,2} \frac{L_2}{M_2} S_{a2} = \mathbf{800 \cdot 1.0} \cdot (-0.174) \cdot 0.114 \cdot 10 = -158.69\ kN$$

The second modal shape contribution to the shear base seismic force is thus:

$$V_{02} = \sum F_{i2} = 69.82\ kN$$

Combining the results for the two modal shapes per each storey, we finally get:

$$F_1 = \sqrt{F_{11}^2 + F_{12}^2} = \sqrt{602.80^2 + 204.71^2} = 636.61\ kN$$

$$F_2 = \sqrt{F_{21}^2 + F_{22}^2} = \sqrt{1038.15^2 + 99.97^2} = 1042.95\ kN$$

$$F_3 = \sqrt{F_{31}^2 + F_{32}^2} = \sqrt{1473.50^2 + 76.17^2} = 1475.47\ kN$$

$$F_4 = \sqrt{F_{41}^2 + F_{42}^2} = \sqrt{1116.29^2 + 158.69^2} = 1127.51\ kN$$

which totally give a shear base seismic force of **V_0 = 4282.54 kN**.

Following the previous procedure, we find the following shear forces for each storey:

V_4 = F_4 = **1127.51 kN**

V_3 = V_4 + F_3 = 1127.51 + 1475.47 = **2602.98 kN**

V_2 = V_3 + F_2 = 2602.98 + 1042.95 = **3645.93 kN**

V_1 = V_2 + F_1 = 3645.93 + 636.61 = **4282.54 kN**

As a result the corresponding values for bending moments, are:

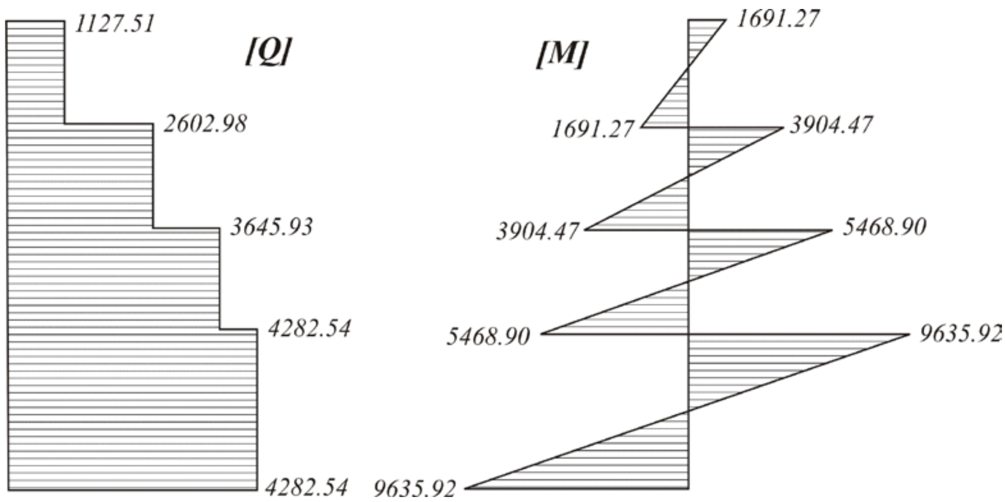
$\mathbf{M_4} = V_4 \cdot h_4/2 = 1127.51 \cdot 3/2 = \mathbf{1691.27\ kNm}$,

$\mathbf{M_3} = V_3 \cdot h_3/2 = 2602.98 \cdot 3/2 = \mathbf{3904.47\ kNm}$,

$\mathbf{M_2} = V_2 \cdot h_2/2 = 3645.93 \cdot 3/2 = \mathbf{5468.90\ kNm}$,

$\mathbf{M_1} = V_1 \cdot h_1/2 = 4282.54 \cdot 4.5/2 = \mathbf{9635.92\ kNm}$,

Following are the corresponding shear force and bending moment diagrams.



Comparing the results of two methods, especially the [Q] and [M] diagrams, it is obvious that values coming from the modal superposition (dynamic) method are from 10 to 20% smaller than those coming from the equivalent static method.

The dynamic method, although time consuming and sophisticated, seems to be closer to reality and this may be an additional reason to be used by computers.

On the other hand, the simplified static method, presenting a simplicity, provides results that are safer for the construction, although less economical.

Exercise 13

The design flexural capacities of beams for the frame structure depicted in Fig. 1 are given next to the corresponding tension side of each joint (top or bottom). Calculate the minimum design flexural capacity of the columns to fulfil the capacity design conditions.

Data and assumptions

- The seismic action controls the design of beams, i.e. $M_{Rd} = M_{Eb}$,
- Columns have symmetric sections and reinforcement and $\gamma_{Rd} = 1.4$,
- The greater axial load below the joint of a column, increases its flexural capacity by 15 % compared to that above.

For the same frame, if the columns' flexural capacities are depicted in Fig. 2, indicate where the plastic hinges will form, for a seismic action from right to left.

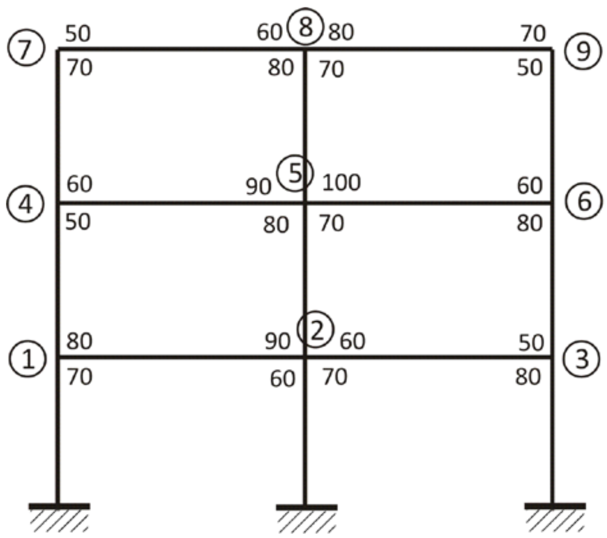


Fig. 1
BEAMS flexural capacity

Seismic action

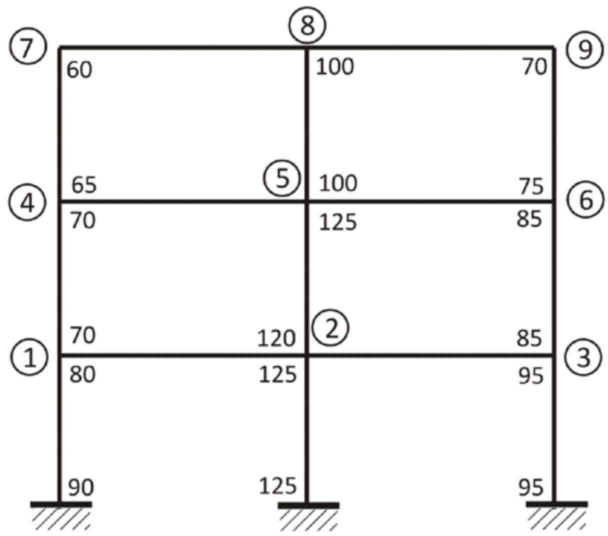


Fig. 2
COLUMNS flexural capacity

Solution

For a column to fulfill the capacity design conditions, the minimum design flexural capacity, $M_{CD,C}$ must be:

$$M_{CD,C} = \alpha_{CD} \cdot M_{EC}$$

where $M_{CD,C}$ is the flexural capacity of the column, M_{EC} is the bending moment of column, derived from seismic analysis and α_{CD} the joint capacity magnification factor, yielding from the equation

$$\boldsymbol{\alpha_{CD}} = \boldsymbol{\gamma_{RD}} \frac{\sum \boldsymbol{M_{Rd}}}{\sum \boldsymbol{M_{Eb}}}$$

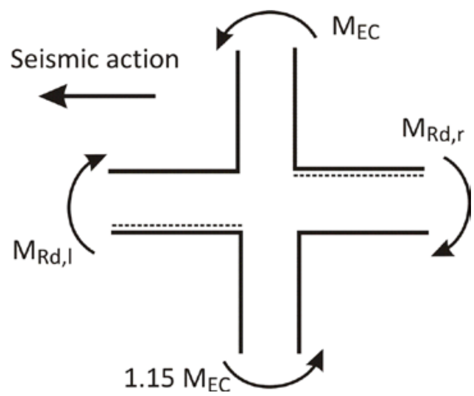
where ΣM_{Rd} is the sum of the beams' flexural capacities gathered on the joint as a result of the column's bending moment and ΣM_{Eb} the corresponding sum of the beams' seismic moments, derived from the analysis, following always the same direction to generate M_{EC}.

In our case, it is: $M_{Rd} = M_{Eb}$.

Therefore $\alpha_{CD} = \gamma_{RD} = 1.4$ and $\mathbf{M_{CD,C} = 1.4 \cdot M_{EC}}$.

Since the seismic action is from right to left, it follows that joints tend to turn leftwards; the beams, thus, reacting to this rotation, tend to turn rightwards. The values of beams' bending moments, to be taken into account from both sides of the joint, are therefore the <u>lower left</u> and the <u>upper right</u> (tensional sides).

The equilibrium of a typical joint, excluding those of the upper storey, gives:

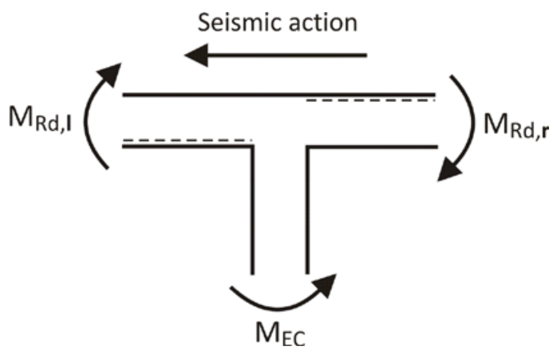


$$\sum M_{Rd} + \sum M_{EC} = 0$$

$$\rightarrow \quad M_{Rd,l} + M_{Rd,r} = M_{EC} + 1.15 M_{EC} = 2.15 M_{EC}$$

$$\rightarrow \quad M_{EC} = \frac{M_{Rd,l} + M_{Rd,r}}{2.15}$$

Capacity design is not compulsory for the upper storey. The columns' flexural capacities are simply derived from the corresponding joint equilibrium, i.e.



$$\sum M_{Rd} + \sum M_{EC} = 0$$

$$\rightarrow \quad M_{Rd,l} + M_{Rd,r} = M_{EC}$$

The following table provides the minimum flexural capacities for all the columns, upon and below of each joint.

Joint **Number**	**M**Rd,**l**	**M**Rd,**r**	**M**CD,C$_{req}$$^{\textbf{above}}$	**M**CD,C$_{req}$$^{\textbf{below}}$
1		80	52.09	59.9
2	60	60	78.14	89.86
3	80		52.09	59.9
4		60	39.07	44.93
5	80	100	117.21	134,79
6	80		52.09	59.9
7		50		50,0
8	80	80		160,0
9	50			50,0

The procedure followed for joint 5, for instance, is:

$M_{Rd,l}$ = 80 kNm, $M_{Rd,r}$ = 100 kNm

The required flexural capacities (bending moments) above and below the joint are therefore:

$$M_{CD,C,req}^{\textbf{above}} = 1.4\cdot(80+100)/2.15 = 117.21 \text{ kNm},$$

$$M_{CD,C,req}^{\textbf{below}} = 1.15\cdot M_{CD,C,req}^{above} = 134.79 \text{ kNm}$$

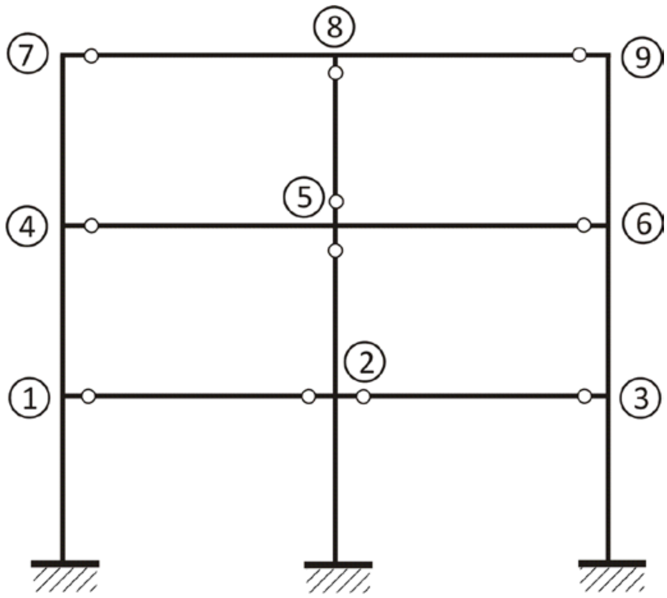
For the beams' flexural capacities shown in Fig. 1 and for the columns' flexural capacities displayed in Fig. 2, the plastic hinges to be formed are depicted in the following figure with a circle.

For instance, the **required** columns' flexural capacities above and below joint **5** are:

$$M_{CD,C,req}^{above} = 117.21 \text{ kNm}, \quad M_{CD,C,req}^{below} = 134.79 \text{ kNm},$$

while the corresponding **actual** capacities for the same joint are 100 and 125 kNm.

Since 100 < 117.21 and 125 < 134.79, it yields that columns <u>**are not strong enough**</u>, and, therefore, the plastic hinges, will be formed at the **columns** themselves.



Depiction of plastic hinges formed on the frame structure

Exercise 14

The water tower of Fig. 1, the elastic design spectrum of which is depicted in Fig. 2, has been constructed according to the Greek Seismic Code EAK 2000 for a behavior factor $q = 3.3$, a seismic risk zone I ($A = 0.16g$), soil category B and Importance factor Σ2 ($\gamma = 1.0$).

A) The water tower, presenting a total weight (self-weight and water) 1200 kN and a Natural period $T = 0.7$ sec, rests on 4 similar columns. For $g = 10\ m/sec^2$, **calculate**:

1. The design seismic force and the corresponding shear force and bending moment which is developed at the base of each column.
2. The expected relative displacement of water tower in the case of an earthquake.

B) After the construction of tower an earthquake occurred, the elastic response spectrum of which is illustrated in Fig. 3.

Considering that the real horizontal force P_{real} for which yielding of columns is initiating is 30% greater than the corresponding design force, calculate:

1. The ductility developed during the earthquake.
2. The maximum shear force at each column.
3. The maximum relative displacement of the tower during the earthquake.
4. The maximum acceleration recorded by an accelerograph, laid on the water tower.
5. Do you think the water tower had reached the risk of collapse during the earthquake?

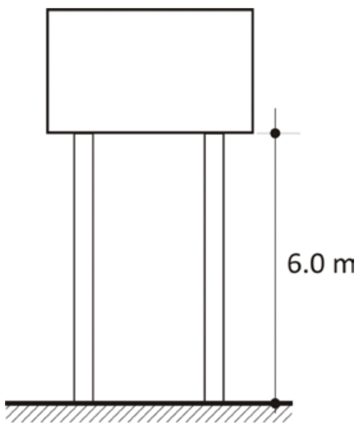


Fig. 1

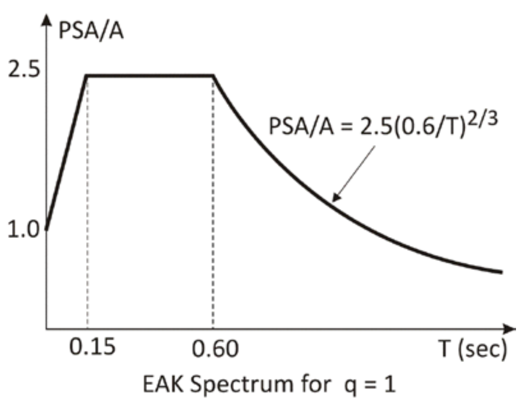


Fig. 2

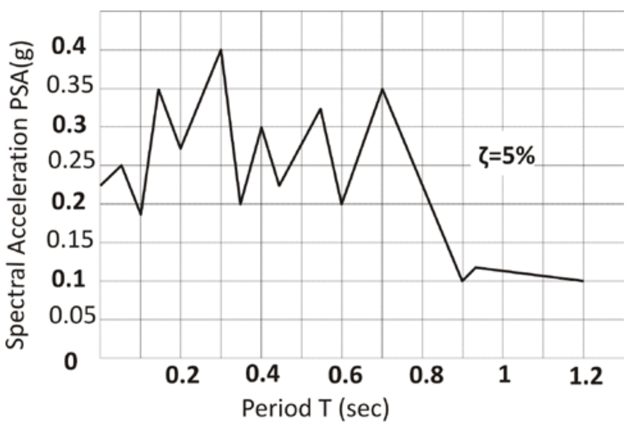


Fig. 3

Solution

A) The elastic design seismic force demands the corresponding horizontal seismic force, which will be derived through the design spectrum.

1) Since the natural period of the structure is 0.7 sec, we are obviously on the third branch of the design spectrum; therefore

$$\frac{PSA}{A} = 2.5\left(\frac{0.6}{T}\right)^{2/3} = 2.5\left(\frac{0.6}{0.7}\right)^{2/3} = 2.256$$

or PSA = 2.256·A = 2.256·0.16g = 0.36g

Consequently the elastic design horizontal seismic force, P_{el_d}, is

$$P_{el\_d} = \frac{W}{g} \cdot \mathrm{PSA} = \frac{1200}{10} \cdot 0.36 \cdot 10 = 432\ kN$$

For q = 3.3, the design seismic force, P_d, is:

$$\boldsymbol{P_d} = \frac{P_{el\_d}}{q} = \frac{432}{3.3} = \mathbf{130.9}\ \boldsymbol{kN}$$

Therefore, for each column, are:

Design shear force: $\mathbf{V_d}$ = P_d/4 = **32.73 kN**

Design bending moment: $\mathbf{M_d}$ = V_d·h/2 = 54·6/2 = **98.17 kNm**.

2) In the case of an earthquake, the relative displacement, SD, of the water tower can be calculated through the relation

$$PSA = \omega^2 SD \quad \rightarrow \quad SD = \frac{T^2}{4\pi^2} PSA$$

For T = 0.7 sec and PSA = 0.36g, it yields

$$SD = \frac{T^2}{4\pi^2} PSA = \frac{0.7^2}{4\pi^2} 0.36 \cdot 10 = 0.045\ m$$

B) Taking into account the overstrength, developed at columns during the earthquake after the construction of tower, we proceed to the following steps:

1) The ductility demanded during the earthquake is

$$\mu = \frac{\delta_{max}}{\delta_y} = \frac{P_{el}}{P_y}$$

where: δ_{max} is the maximum displacement of the tower

δ_y is the displacement of the tower when first yield is initiating

P_{el} is the elastic horizontal force during the new earthquake, calculated through the response spectrum and

P_y is the yield force, i.e. the force when first yield is initiating.

From the given response spectrum of Fig. 3, for T = 0.7 sec, it yields PSA = 0.35g.

The elastic horizontal force during the new earthquake is therefore

$$P_{el} = m \cdot PSA = 120 \cdot 0.35 \cdot 10 = 420 \text{ kN}.$$

On the other hand, the force P_y, when first yield is initiating, is

$$P_y = 1.3 \cdot P_d = 1.3 \cdot 130.9 = 170.17 \text{ kN}$$

Consequently, the ductility developed during the earthquake is

$$\boldsymbol{\mu} = \frac{P_{el}}{P_y} = \frac{420}{170.17} = \mathbf{2.47}$$

2) The maximum shear force which will appear after the earthquake at each column is obviously the corresponding to each column seismic force when first yield is initiating, i.e.

$$\boldsymbol{maxV} = \frac{1}{4} P_y = \frac{170.17}{4} = \mathbf{42.54\ kN}$$

3) The maximum relative displacement of the water tower during the earthquake will be calculated through a way similar to that used for the corresponding displacement at the design stage. The difference here is that the relative acceleration, PSA, will be derived from the corresponding **response** spectrum.

For T = 0.7 sec, the response spectrum defines PSA = 0.35g. Therefore

$$\boldsymbol{SD} = \frac{T^2}{4\pi^2} PSA = \frac{0.7^2}{4\pi^2} 0.35 \cdot 10 = \mathbf{0.043\ m}$$

4) The maximum possible acceleration, α_{max}, which could be recorded by an accelerograph laid on the water tower, will obviously correspond at the time when yield is initiating at the columns, i.e. when the horizontal seismic force reaches the value of P_y.

In this case, it will be

$$P_y = m \cdot a_{max} \quad \rightarrow \quad \boldsymbol{a_{max}} = \frac{P_y}{m} = \frac{170.17}{120} = \mathbf{1.42\ m/sec^2}$$

5) In order to examine the case if the water tower had reached the risk of collapse, we have to calculate the behavior factor, q_e, developed during the earthquake and compare it with the corresponding behavior factor, q, which has been taken into account during the design phase. Then,

- If $q_e < q$, the structure had not reached the risk of collapse. But
- If $q_e > q$, the structure had already past the risk of collapse.

The behavior factor, q_e, developed during the earthquake, is

$$q_e = \mu \cdot q_0, \quad \text{where}$$

- μ is the ductility factor and
- q_0 is the ratio P_y/P_d, i.e. the overstrength factor.

In our case, it is $\mu = 2.47$ and $q_0 = 1.30$. Therefore

$$\mathbf{q_e} = \mu \cdot q_0 = 2.47 \cdot 1.30 = \mathbf{3.21 < 3.3}$$

Consequently the structure had **not** reached the risk of collapse during the earthquake.

Exercise 15

1. A structure, presenting a weight of 1500 kN, a natural period T = 0.8 sec and a height of 9 m, has been designed against earthquake with a behavior factor q = 3.2. If the maximum horizontal force, carried by the structure, is P_y = 450 kN, calculate:

    a. The corresponding maximum acceleration.

    b. The available overstrength, if the structure has been built for a design seismic force, P_d = 320 kN.

    c. The maximum elastic displacement that the structure can sustain.

    d. The maximum possible displacement, developed without a collapse risk, if the structure is really under a collapse risk for a ductility factor μ = 4.

2. Two structures A and B present the same mass, same height and have been designed with the same behaviour factor, q, and the same design force, P_d.

    a. If the structure A presents triple the stiffness of B, how are the maximum displacements related, according to the design procedure?

    b. If the structures, instead of having the same mass, present the same natural period, while the structure A is designed for 3/4 the ground acceleration than that of structure B, repeat the question 2a.

3. Two structures A and B present the same natural period and have been designed with the same behavior factor, q, and the same design acceleration $\Phi_d(T)$. If structure A presents double stiffness compared with structure B, how are the maximum displacements related, according to the design procedure?

4. Two adjacent structures with same mass and same natural period have been designed according to EAK.

    - The first one, with q = 1 and $\Phi_d(T)$ = 0.748g, was designed on the limit without overstrength.

    - The second, with a q = 3.4, presented some overstrength.

    During a seismic event, the first suffered a significant damage, while a max acceleration 0.32g was recorded on the roof of the second.

    What was the overstrength factor on the second structure?

Solution

1. **a.** For the maximum possible acceleration, a_{max_y}, which will take place at the start of yielding, we obviously take into account the maximum horizontal force that the structure can sustain, P_y, i.e.

$$\boldsymbol{\alpha_{max\_y}} = \frac{P_y}{m} = \frac{450}{1500/10} = 3\, m/sec^2 = \mathbf{0.3}\boldsymbol{g}$$

b. Since the structure has been built for a design seismic force, P_d = 320 kN, it follows that its overstrength is

$$\frac{P_y - P_d}{P_d} = \frac{450 - 320}{320} = 0.4063 = \mathbf{40.63}\%$$

c. For calculating the max elastic displacement we will obviously use the above maximum possible acceleration, i.e.

$$\boldsymbol{\delta_{max\_y}} = \frac{a_{max\_y}}{\omega^2} = \frac{T^2}{4\pi^2} a_{max\_y} = \left(\frac{0.8}{6.28}\right)^2 \cdot 0.3g = \mathbf{0.049}\, \boldsymbol{m}$$

d. Since the structure is really under a collapse risk for a ductility factor μ = 4, having calculated the max elastic displacement, δ_{max_y}, at the start point of yielding, the maximum possible displacement will be derived making use of the ductility factor, i.e.

$$\mu = \frac{\delta_{max}}{\delta_{\max\_y}} \quad \rightarrow \quad \boldsymbol{\delta_{max}} = \mu \cdot \delta_{\max\_y} = 4 \cdot 0.049 = \mathbf{0.196}\, \boldsymbol{m}$$

2. **a.** Since the two structures present the same given design properties, the following relations will hold:

$$\delta^A_{max} = \delta^A_y \cdot q = \frac{P_y}{k_A} q$$

$$\delta^B_{max} = \delta^B_y \cdot q = \frac{P_y}{k_B} q$$

Dividing by parts the above equations, it yields:

$$\frac{\delta^A_{max}}{\delta^B_{max}} = \frac{k_B}{k_A} = \frac{1}{3} \quad \rightarrow \quad \boldsymbol{\delta^A_{max}} = \frac{\mathbf{1}}{\mathbf{3}} \boldsymbol{\delta^B_{max}}$$

b. In this case we have to correlate the maximum displacements with the corresponding maximum accelerations where the frequencies are involved. It is:

$$T_A = T_B \rightarrow \omega_A = \omega_B$$

$$\delta^A_{max} = \delta^A_y \cdot q = \frac{\alpha^A_y}{\omega^2_A} q$$

$$\delta^B_{max} = \delta^B_y \cdot q = \frac{\alpha^B_y}{\omega^2_B} q$$

In the same way, dividing the previous equations by parts, it yields:

$$\frac{\delta_{max}^A}{\delta_{max}^B} = \frac{\alpha_y^A}{\alpha_y^B} = \frac{3}{4} \qquad \rightarrow \qquad \boldsymbol{\delta_{max}^A = \frac{3}{4}\delta_{max}^B}$$

3. Similarly the two structures have been designed with members presenting the same properties, but $k_A = 2k_B$. Making use of the previous equations and taking also into account that the design acceleration, $\Phi_d(T)$, can replace the maximum acceleration divided by q, i.e. $\alpha_{max} = \Phi_d(T) \cdot q$, it holds:

$$T_A = T_B \quad \rightarrow \quad \omega_A = \omega_B$$

$$\delta_{max}^A = \frac{\Phi_d(T)}{\omega_A^2} q$$

$$\delta_{max}^B = \frac{\Phi_d(T)}{\omega_B^2} q$$

and obviously $\boldsymbol{\delta_{max}^A = \delta_{max}^B}$, i.e. independent of stiffness.

4. The structure A has been designed elastically (q = 1) for a limit design acceleration $\Phi_d(T) = 0.748g$, without overstrength.

During the earthquake, it suffered significant damage. Therefore it had already past the point of elastic yielding under the acceleration of 0.748g.

If structure B had also been designed elastically, it would have reached the start point of yielding under the same acceleration, i.e. 0.748g.

However, due to the applied behavior factor q = 3.4, the yield point, according to design, has already been realized for a

$$\Phi_d(T) = 0.748g/3.4 = \mathbf{0.22g}.$$

Consequently, since on the structure B, an acceleration of 0.32g has been recorded, it follows that we are already in the yielding stage and hence, the overstrength factor is:

$$\frac{P_y}{P_d} = \frac{m \cdot a_y}{m \cdot a_d} = \frac{0.32g}{0.22g} = \mathbf{1.45}$$