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A RATIONAL APPROACH ON THE STUDY OF A BIAXIAL BUCKLING MODEL

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Abstract. *The nature of biaxial buckling and the quantitative calculation of different parameters which are involved on the load carrying capacity of columns is the aim of this project. Columns with comparable strength about both axes will experience an interaction between various buckling parameters about both axes. Current methodologies for the design of columns in compression or a combined action of it with bending, assume failure occurs when the column buckles about the minor axis of its cross section. If the buckling load about the major axis is significantly higher than that required to initiate instability about the minor axis, then a situation arises at failure, where, much of the buckling resistance about the major axis remains unutilized. An efficient designer, would therefore aim to produce simultaneity of buckling about both axes. Structural engineers normally pay considerable attention to the selection of sections and arrangement of bracing to achieve such situations. The target of the project is therefore, to investigate the development of a theoretical model of biaxial buckling, that will be used to evaluate upper and lower bounds to the collapse load.*

1 INTRODUCTION

In a previous discussion [1] of uniaxial buckling, it was established that the imperfection approach is the only rational analysis which can be used to predict accurately the strength of a uniaxial buckling of columns. According to this analysis, the column strength in a given direction was found to be a function of the Euler critical load and the total equivalent imperfection.

For a given cross sectional area, an optimum column design, would require that:

- The critical load is maximized.
- The total equivalent imperfection is minimized.
- The buckling strength is the same in all directions.

The first requirement is dealt with standard structural mechanics text book whilst the second can be achieved by improvements in the manufacturing process, the quality of workmanship and loading arrangement in the structure.

The third requirement however, concerning the simultaneity of buckling, is the main subject of this project.

Whilst it might be impossible to achieve simultaneity of buckling in all directions, it is nevertheless desirable to design the columns so that the difference between the maximum and minimum buckling strength is minimized.

This approach to column design, will, inevitably poses basic, yet essential questions about the interaction between various buckling parameters. In the rest an attempt is made to find rational answers to some of them.

2 NOTATION

E	Young's Modulus.
M	Bending moment.
P	Applied axial load.
P_c	Critical load of elastic buckling.
P_{ci}	Euler critical load at the ith mode.
P_E	Euler critical load.
P_b	Buckling load.
P_y	Squash load.

P_{fy} Load corresponding to the first yield.
P_{fh} Load corresponding to the first hinge.
ρ Imperfection parameter.
ξ_0 Total equivalent imperfection.
A Cross sectional area.
Z Plastic modulus of the section.
I Cross sectional second moment of area.
i Mode index, i.e. 1, 2, 3…
Φ Characteristic shape function.
L_e Effective length.
w Central deflection.
w_i Maximum deflection at the i^{th} mode.
w_0 Amplitude factor associated with geometrical imperfections.
w^p Amplitude factor associated with proportional loading imperfections.
w^n Amplitude factor associated with non proportional loading imperfections.
σ_m Maximum stress on the cross section.
m Moment ratio factor.
m^p Moment induced by proportional loading.
m^n Moment induced by non-proportional loading.
M Resultant bending moment.
M_p Plastic moment of resistance.
M^l Linear moment component.
u Deflection in the x-direction.
v Deflection in the y-direction.
b, d Cross sectional dimensions.
α, β Curvature functions. Depths of tensile stress blocks.
x_o, y_o Coordinates of center of gravity.

3 THEORETICAL BACKGROUND

3.1 Failure criterion

In order to determine the strength of a column buckling biaxially, it is necessary to define a failure criterion at which the column will be considered to have failed. For this purpose, the two limiting cases given below are adopted,

i. First Yield Formation, where the column will be deemed to have failed if the stress at some point in a cross section reaches the yield value. The load that causes the initiation of yielding in the column, is a lower bound to the collapse load.

ii. First Hinge Formation, where the column will be deemed to have failed if the stress in a cross section, has reached the yield value everywhere. The load that causes a plastic hinge to form in the column is an upper bound to the collapse load.

3.2 First Yield Analysis

Fig. 1 shows a column with total equivalent imperfection $\xi_x(z)$ and $\xi_y(z)$ in the x and y direction respectively.

The differential equations of equilibrium are:

$$EI_{yy}\left[u^{iv}(z)-\xi_x^{iv}(z)\right]+Pu''(z)=0 \tag{1}$$

$$EI_{xx}\left[v^{iv}(z)-\xi_y^{iv}(z)\right]+Pv''(z)=0 \tag{2}$$

We can express the deflections u, v along with the total equivalent imperfections ξ_x and ξ_y as series in the characteristic functions associated with the i^{th} buckling mode in the x and y directions respectively, Φ_i and Ψ_i, such that,

$$u(z)=\sum_{i=1}^{\infty}u_i\Phi_i(z) \quad \text{and} \quad \xi_x(z)=\sum_{i=1}^{\infty}\xi_{xi}\Phi_i(z) \tag{3}$$

$$v(z)=\sum_{i=1}^{\infty}v_i\Psi_i(z) \quad \text{and} \quad \xi_y(z)=\sum_{i=1}^{\infty}\xi_{yi}\Psi_i(z) \tag{4}$$

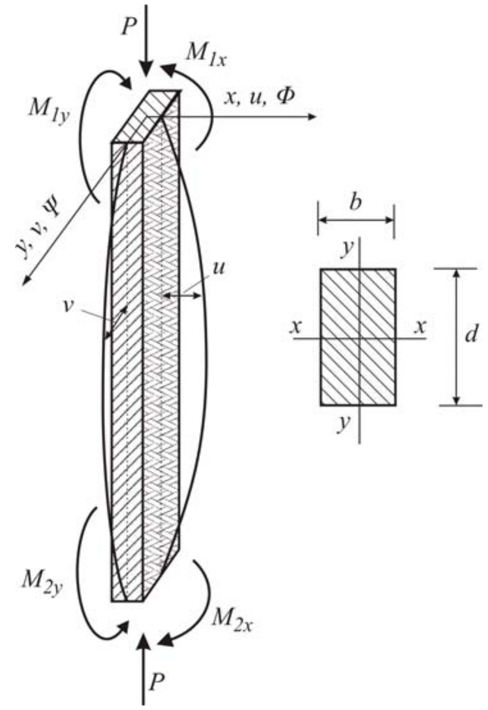

Figure 1: Biaxial buckling of column.

At any arbitrary cross section along the column length, the stress distribution is the resultant of three stress components shown in Fig. 2, namely, the direct compressive stress, resulting from the axial compressive load and two stress components resulting from non linear moments M_x and M_y, induced by the imperfections in the column. In addition to the three stress components, the effect of any linear moments acting on the section must be added, to calculate the maximum stress in the section, which therefore, is given by

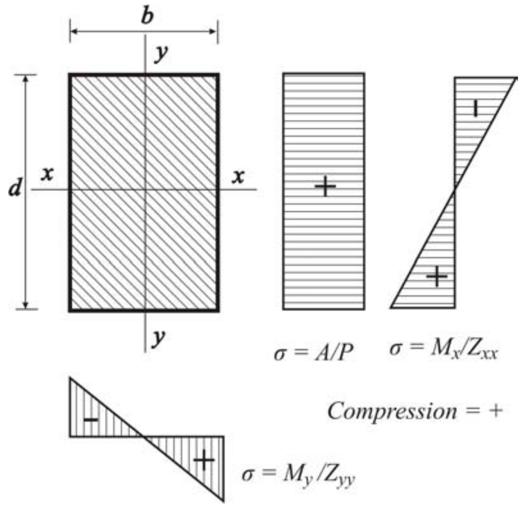

Figure 2: Stress distribution of cross section

$$\sigma_m = \frac{P}{A} \pm \frac{M_x}{Z_{yy}} \pm \frac{M_y}{Z_{xx}} \pm \frac{M_x^l}{Z_{yy}} \pm \frac{M_y^l}{Z_{xx}} \tag{5}$$

where M^l refers to linear moment components.

Using the expression (45) for the non linear bending moment, given in reference [1], equation (5) becomes,

$$\sigma_m = \frac{P}{A} \pm \frac{EI_{yy}}{Z_{yy}} \sum_{i=1}^{\infty} \frac{P}{P_{cxi} - P} \xi_{xi} \Phi_i^{"}(z) \pm \frac{EI_{xx}}{Z_{xx}} \sum_{i=1}^{\infty} \frac{P}{P_{cyi} - P} \xi_{yi} \Psi_i^{"}(z) \pm \frac{M_x^l(z)}{Z_{yy}} \pm \frac{M_y^l(z)}{Z_{xx}} \tag{6}$$

The column will be assumed to have failed when $\sigma_m = \sigma_y$ and a lower bound to the collapse load P_b is given by the solution of the equation,

$$\frac{P_y}{A} = \frac{P_b}{A} \pm \frac{EI_{yy}}{Z_{yy}} \sum_{i=1}^{\infty} \frac{P_b}{P_{cxi} - P_b} \xi_{xi} \Phi_i''(z) \pm \frac{EI_{xx}}{Z_{xx}} \sum_{i=1}^{\infty} \frac{P_b}{P_{cyi} - P_b} \xi_{yi} \Psi_i''(z) \pm \frac{M_x^l(z)}{Z_{yy}} \pm \frac{M_y^l(z)}{Z_{xx}} \tag{7}$$

Let us now consider the special case of a column with symmetrical end conditions in each direction (boundary conditions need not be the same for both directions), where, the contribution of the 1st mode of buckling to the non linear moment is dominant. In a further simplification, we will assume that no linear moments act on the column.

The maximum bending moment in this case, is at the center of the column length. The maximum stress in the column is obtained by using the expression for bending moment given in equation (48) [1], and the lower bound to the buckling load P_b is

$$\frac{P_y}{A} = \frac{P_b}{A} + \frac{P_b P_{cx1}}{P_{cx1} - P_b} \cdot \frac{\xi_{x1} \cdot a}{Z_{yy}} + \frac{P_b P_{cy1}}{P_{cy1} - P_b} \cdot \frac{\xi_{y1} \cdot \beta}{Z_{xx}} \tag{8}$$

where:

P_{cx1}, P_{cy1}: Are the Euler critical load associated with the 1st mode in the *x* and *y* direction respectively

ξ_{x1}, ξ_{y1}: Are the 1st mode amplitudes of the total equivalent imperfection in the *x* and *y* direction respectively

$$a = \frac{1}{1 - \cos\left(\dfrac{\pi L_x}{2L_{ex1}}\right)} \quad \text{and} \quad \beta = \frac{1}{1 - \cos\left(\dfrac{\pi L_y}{2L_{ey1}}\right)} \tag{8a}$$

are curvature functions, resulting from Eq. (50) [1].

Writing equation (8) in non dimensional form, and rearranging we obtain a cubic equation in *p*,

$$p^3 - p^2\left(p_{cx}\rho_x a + p_{cy}\rho_y \beta + p_{cx} + p_{cy} + 1\right) + p\left[p_{cx}p_{cy}\left(\rho_x a + \rho_y \beta + 1\right) + p_{cx} + p_{cy}\right] - p_{cx}p_{cy} = 0 \tag{9}$$

where

$p = \dfrac{P_b}{P_y}$, is the non dimensional buckling load

$p_{cx} = \dfrac{P_{cx1}}{P_y}$ and $p_{cy} = \dfrac{P_{cy1}}{P_y}$ are non dimensional Euler critical loads, whereas

$\rho_x = \xi_{x1}\dfrac{A}{Z_{yy}}$ and $\rho_y = \xi_{y1}\dfrac{A}{Z_{xx}}$ are non dimensional imperfection parameters.

For a given column, the Euler critical loads and the imperfection parameters are known (or assumed), therefore equation (9) can be solved by iterative techniques for the non dimensional buckling load of the column.

Equation (9) is independent of the shape of the cross section and can be applied to all common sections (I,H, Hollow and Solid Boxes).

When the column is acted upon by linear bending moments, whose maximum value is at the column center, the stresses caused by the linear moments must be added to the R.H.S of equation (8). In the case where the maximum linear moments occur away from the column center, then it becomes necessary to include the contributions of higher modes to the non linear moments [2]. The maximum stress at different sections along the column length must be evaluated and a lower bound to the buckling load can be established accordingly. In the majority of cases, it is sufficient to include the contributions of the first two modes when calculating the non linear moments.

A computer program that solves equation (9) using a Newton Raphson routine has been developed. The program is intended to establish the extent of interaction between the imperfection parameters, and it therefore, produces charts that relate the buckling strength to various combinations of total equivalent imperfections in the *x* and *y* axes.

The different curves in the chart represent successive vertical planes through a failure surface which have the non dimensional buckling load as a vertical axis *(z)*, and the non dimensional imperfection parameters, as horizontal axes *(x,y).*

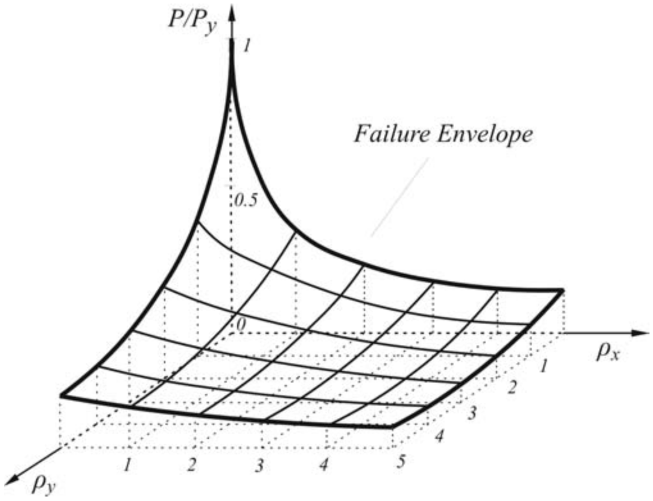

Figure 3: Imperfection sensitivity failure surface.

The result indicates the existence of significant interaction between imperfections about both axes, leading to a reduction in the buckling load. This reduction is quantified by the vertical spacing of the curves. It is clear that the reduction in the buckling load is increased as the difference between P_{cx} and P_{cy} decreases. The maximum reduction corresponds to $P_{cx} = P_{cy}$.

3.3 Plastic collapse of biaxially buckling columns

3.3.1 Elastic plastic interaction

The next step in this theoretical development is to define a plastic mechanism by which plastic failure occurs in biaxially buckling columns.

The generalized Ayrton – Perry imperfection approach discussed in reference [1], has defined correctly, the non linear response of imperfect columns. In section 3.2, this approach has been extended to cover the case of two dimensional buckling, and concluded that for columns which are imperfect with respect to two axes, there will be two sets of bending moments about the minor and major axis of the column, both of which are non linearly related to the axial load.

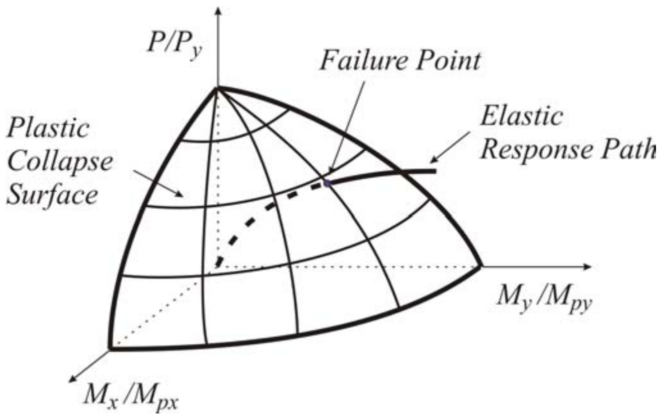

Figure 4: Plastic collapse surface

Let us now define a three dimensional coordinate system, in which the vertical axis *(z)* represents a non dimensional axial load, and the two horizontal axes *(x,y)* represent generalized non dimensional bending moments about the major and minor axis of the column respectively. The path in space, followed by a point representing the above three values of a given column, as the axial load is increased, is referred as the *Elastic Non Linear Path*. This path is a function of the axial load, the Euler critical loads about the major and minor axis, the boundary conditions and the imperfection parameters in the *x* and *y* direction.

For a given cross section of the column, there exist a plastic collapse surface that represent *all* possible combinations of axial load and bending moments that can exist in equilibrium with a fully plastic section.

The point where the elastic non linear path penetrates the plastic collapse surface is an upper bound to the collapse load.

3.3.2 Plastic Collapse Surface

Fig. 5 shows the cross section of a column subjected to an axial load P, and bending moments M_x and M_y acting about the x and y axis respectively. The resultant moment $M(R)$, acts about an axis which forms an angle θ with the x axis.

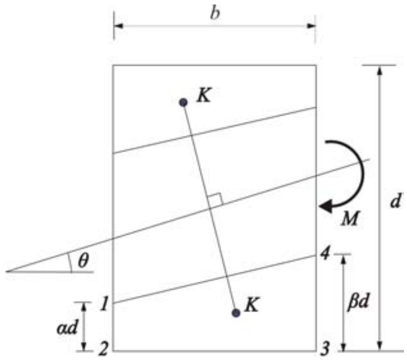

Figure 5: Trapezium stress block

Three distinct cases [3] of plastic equilibrium can occur and these are analyzed below :

Case (a): Trapezium Stress Block

If the column reaches a state of full plasticity, i.e the stresses have reached σ_y everywhere in the cross section, then equilibrium considerations suggest that the column will have a stress distribution similar to that shown in Fig. 5 with the central block in compression, the upper block in tension and the lower block in compression, such that the latter two blocks constitute a couple equal to the applied resultant moment $M(R)$. The main feature of this stress distribution is the assumption, that, lines separating the central compression core and the upper tension zone is not necessarily parallel to the axis of resultant moment.

From the geometry of the section it can be seen that the coordinates of the center of gravity of the lower compression block, (x_o, y_o), are given by

$$x_o = \frac{(a+2\beta)}{3(a+\beta)} \cdot b \quad \text{and} \quad y_o = \frac{\left(a^2 + a\beta + \beta^2\right)}{3(a+\beta)} \cdot d \tag{10}$$

Since there are no torsional moments applied, the centers of gravity of the lower and upper stress blocks must lie in a straight line *NORMAL* to the axis of the resultant moment $M(R)$, passing through the C.G. of the cross section. This condition is referred as the *Normality Condition* .

The slope of this Normal is $-cot(\theta)$; if we put $m = tan(\theta)$, then the equation of the *Normal* is

$$y - \frac{d}{2} = -\frac{1}{m}\left(x - \frac{b}{2}\right) \tag{11}$$

The block's center of gravity lies in this line and, therefore, (x_o, y_o) must satisfy equation (11), i.e.

$$y_o - \frac{d}{2} = -\frac{1}{m}\left(x_o - \frac{b}{2}\right) \tag{12}$$

or, substituting for x_o and y_o from (10),

$$\frac{(a^2 + a\beta + \beta^2)}{3(a+\beta)} \cdot d - \frac{d}{2} = -\frac{1}{m} \cdot \frac{(a+2\beta)b}{3(a+\beta)} - \frac{b}{2} \tag{13}$$

After simplifying, and putting $n = b/d$, equation (13) becomes

$$a^2 + \left(\beta - \frac{3}{2} - \frac{n}{2m}\right)a + \left[\beta^2 - \frac{3\beta}{2} + \frac{n\beta}{2m}\right] = 0 \tag{14}$$

Equation (14) defines the necessary condition for the section not to experience torsional moments.

Equilibrium of axial forces of the column gives

$$P = \sigma_y bd\left[1 - (a+\beta)\right] \tag{15}$$

Taking moments about the C.G of the upper tension block we obtain

$$M = \sigma_y \frac{bd}{2}(a+\beta)\cdot 2\cdot\sqrt{\left(\frac{b}{2}-x_o\right)^2+\left(\frac{d}{2}-y_o\right)^2} \tag{16}$$

Taking also into account the relations

$$m=\tan(\theta)=\frac{M_y}{M_x}\,,\quad n=\frac{b}{d}\,,\quad M=M_x{}^2+M_y{}^2$$

the component moments M_x and M_y can be expressed by the equations

$$M_x=\frac{\sigma_y}{\sqrt{1+m^2}}bd(a+\beta)\cdot\sqrt{(nd)^2\left[\frac{1}{2}-\frac{(a+2\beta)}{3(a+\beta)}\right]^2+d^2\left[\frac{1}{2}-\frac{\left(a^2+\beta^2+a\beta\right)}{3(a+\beta)}\right]^2} \tag{17}$$

and

$$M_y=\frac{\sigma_y}{\sqrt{1+\frac{1}{m^2}}}bd(a+\beta)\cdot\sqrt{b^2\left[\frac{1}{2}-\frac{(a+2\beta)}{3(a+\beta)}\right]^2+\left(\frac{b}{n}\right)^2\left[\frac{1}{2}-\frac{\left(a^2+\beta^2+a\beta\right)}{3(a+\beta)}\right]^2} \tag{18}$$

Equations (15), (17) and (18) can now be written in a non-dimensional form as

$$\frac{P}{P_y}=1-(a+\beta) \tag{19}$$

$$\frac{M_x}{M_{px}}=\frac{4(a+\beta)}{\sqrt{1+m^2}}\cdot\sqrt{n^2\left[\frac{1}{2}-\frac{(a+2\beta)}{3(a+\beta)}\right]^2+\left[\frac{1}{2}-\frac{\left(a^2+\beta^2+a\beta\right)}{3(a+\beta)}\right]^2} \tag{20}$$

$$\frac{M_y}{M_{py}}=\frac{4(a+\beta)}{\sqrt{1+\frac{1}{m^2}}}\cdot\sqrt{\left[\frac{1}{2}-\frac{(a+2\beta)}{3(a+\beta)}\right]^2+\left(\frac{1}{n}\right)^2\left[\frac{1}{2}-\frac{\left(a^2+\beta^2+a\beta\right)}{3(a+\beta)}\right]^2} \tag{21}$$

where $P_y=\sigma_y bd$ is the squash load, and

$$M_{px}=\sigma_x\frac{bd^2}{4}\quad\text{and}\quad M_{py}=\sigma_y\frac{bd^2}{4}$$

are the plastic moments of the cross section in the *x* and *y* directions respectively.

Case (b): Triangular Stress Block

When the ratio $m = M_y/M_x$ of the applied bending moments is high, the section, reaching the state of full plasticity, develops a triangular stress block, as shown in Fig. 6.

In this Figure, the coordinates of the center of gravity of the triangular section (X_o, Y_o) are respectively

$$X_o=\left(1-\frac{a}{3}\right)b\quad\text{and}\quad Y_o=\frac{\beta}{3}\cdot d \tag{22}$$

The equation of the *NORMAL* to the axis of the resultant Moment is

$$y-\frac{\beta}{3}d\quad=\quad-\frac{1}{m}\left[x-\left(1-\frac{a}{3}\right)b\right] \tag{23}$$

Since this normal passes through the centroid of the section, the point (*b/2, d/2*) must satisfy its equation.

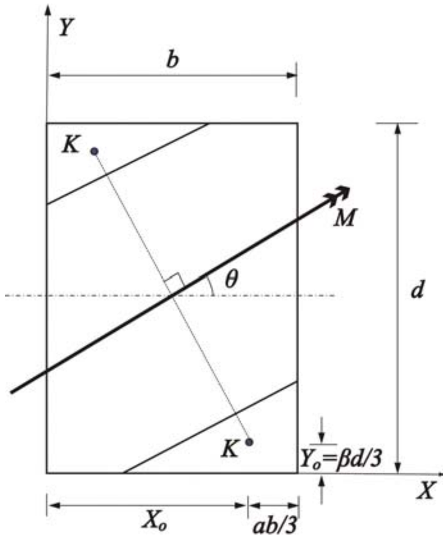

Figure 6: Triangular stress block

$$\frac{d}{2}-\frac{\beta}{3}d \quad = \quad -\frac{1}{m}\left[\frac{b}{2}-\left(1-\frac{a}{3}\right)b\right] \tag{24}$$

Substituting $n = b/d$ and simplifying, yields

$$a=-\frac{3m}{n}\left(\frac{1}{2}-\frac{\beta}{3}\right)+\frac{3}{2} \tag{25}$$

where $0 \le (a,\beta) \le 1$.

Considering equilibrium conditions similar to case (a), we obtain:
Equilibrium of axial forces

$$\frac{P}{P_y}=1-a\beta \tag{26}$$

Moments about C.G. of upper stress block

$$M=\sigma_y\frac{a\beta}{2}bd\cdot 2\cdot\sqrt{d^2\left(\frac{1}{2}-\frac{\beta}{3}\right)^2+b^2\left(\frac{1}{2}-\frac{a}{3}\right)^2} \tag{27}$$

from where eventually the non-dimensional moments M_x and M_y are obtained

$$\frac{M_x}{M_{px}}=\frac{4a\beta}{\sqrt{1+m^2}}\cdot\sqrt{\left[\frac{1}{2}-\frac{\beta}{3}\right]^2+n^2\left[\frac{1}{2}-\frac{a}{3}\right]^2} \tag{28}$$

$$\frac{M_y}{M_{py}}=\frac{4a\beta}{\sqrt{1+\frac{1}{m^2}}}\cdot\sqrt{\frac{1}{n^2}\left[\frac{1}{2}-\frac{\beta}{3}\right]^2+\left[\frac{1}{2}-\frac{a}{3}\right]^2} \tag{29}$$

Case (c): Reversed Trapezium Stress Block

If we keep on increasing the bending moment ratio $m = M_y/M_x$, then the section stress distribution takes the shape of a reversed trapezium block like the one shown in Fig. 7.

The coordinates of the center of gravity, are in this case

$$x_o=\left(1-\frac{a^2+a\beta+\beta^2}{3(a+\beta)}\right)b \quad \text{and} \quad y_o=\frac{a+2\beta}{3(a+\beta)}\cdot d \tag{30}$$

From the normality condition and the fact that the normal passes through (b/2, d/2) we obtain

$$\frac{d}{2}-\frac{a+2\beta}{3(a+\beta)}\cdot d \quad = \quad -\frac{1}{m}\left[\frac{b}{2}-\left(1-\frac{a^2+a\beta+\beta^2}{3(a+\beta)}\right)b\right] \tag{31}$$

Substituting $n = b/d$ and simplifying, yields

$$a^2+\left(\beta-\frac{3}{2}-\frac{m}{2n}\right)a+\left[\beta^2-\frac{3}{2}\beta-\frac{m}{2n}\beta\right] \quad = \quad 0 \tag{32}$$

A similar procedure for forces and moments equilibrium gives the following equations

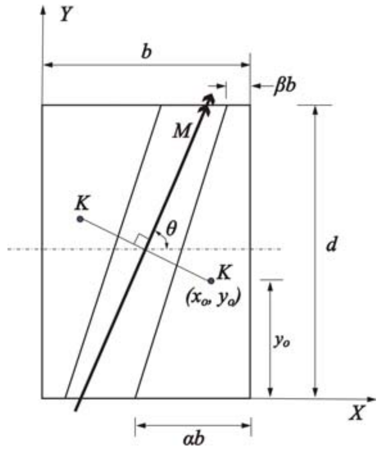

Figure 7: Reversed trapezium stress block

$$\frac{P}{P_y}=1-(a+\beta) \tag{33}$$

$$M=\sigma_y\frac{bd}{2}(a+\beta)\cdot 2\cdot\sqrt{\left(\frac{b}{2}-x_o\right)^2+\left(\frac{d}{2}-y_o\right)^2} \tag{34}$$

$$M_x=\frac{\sigma_y}{\sqrt{1+m^2}}bd(a+\beta)\cdot\sqrt{d^2\left[\frac{1}{2}-\frac{(a+2\beta)}{3(a+\beta)}\right]^2+(nd)^2\left[\frac{1}{2}-\frac{\left(a^2+\beta^2+a\beta\right)}{3(a+\beta)}\right]^2} \tag{35}$$

$$M_y=\frac{\sigma_y}{\sqrt{1+\frac{1}{m^2}}}bd(a+\beta)\cdot\sqrt{\left(\frac{b}{n}\right)^2\left[\frac{1}{2}-\frac{(a+2\beta)}{3(a+\beta)}\right]^2+b^2\left[\frac{1}{2}-\frac{\left(a^2+\beta^2+a\beta\right)}{3(a+\beta)}\right]^2} \tag{36}$$

which, in non-dimensional form, are expressed as

$$\frac{M_x}{M_{px}}=\frac{4(a+\beta)}{\sqrt{1+m^2}}\cdot\sqrt{n^2\left[\frac{1}{2}-\frac{(a+2\beta)}{3(a+\beta)}\right]^2+\left[\frac{1}{2}-\frac{\left(a^2+\beta^2+a\beta\right)}{3(a+\beta)}\right]^2} \tag{37}$$

$$\frac{M_y}{M_{py}}=\frac{4(a+\beta)}{\sqrt{1+\frac{1}{m^2}}}\cdot\sqrt{\left[\frac{1}{2}-\frac{(a+2\beta)}{3(a+\beta)}\right]^2+\left(\frac{1}{n}\right)^2\left[\frac{1}{2}-\frac{\left(a^2+\beta^2+a\beta\right)}{3(a+\beta)}\right]^2} \tag{38}$$

respectively. It should be noted that these equilibrium equations are similar to those obtained in case (a), with b and d interchanged, hence the term *Reversed* Trapezium block.

For a given value of $n = (b/d)$, moment ratio $m = (M_y/M_x)$ and load factor $p = P/P_y$, the Normality equations (14), (25) and (32) along with the Force equilibrium equations (19), (26) and (33), constitute, for each case of stress block separately, a pair (set) of simultaneous second order equations in α and β. Solving these three sets for α and β and choosing the set that satisfies the geometric conditions given in paragraph 4, the corresponding bending moments about both axes can be calculated.

The Normality equation can be expressed as a function $f(\alpha,\beta,n,m) = 0$ and

The Force Equilibrium equation is a function $g(\alpha,\beta,p) = 0$.

The bending moment corresponding to the calculated values of α and β is an Upper bound solution to the collapse combination.

By changing the value of $m = (M_y/M_x)$, and then, for each new value of m, changing the value of non dimensional load $p=P/P_y$, a series of upper bound combinations, between loads and moments, can be calculated and plotted to represent the upper bound envelop of failure.

These calculations are best performed by a computer program having the following algorithm:

1. Select a value of $n = b/d$ envelope
2. Set an initial value of $m = M_y/M_x$. Note that x is the major bending axis
3. Initialize the non-dimensional load factor p to $p = 1$, solve the *Normality* equations and the *Force* equilibrium equations for each one of the cases (a), (b) and (c) to find α and β. Eventually, 3 maximum pairs of values for α and β will be obtained.
4. Those sets of (α, β), which satisfy the geometrical conditions stated below, are then used to calculate the bending moments. The smallest moment - in the event that more than one set of (α, β) satisfy the geometrical conditions - is the upper bound solution to the collapse moment. The geometrical conditions are :

   (i) For cases (a) and (c), (Trapezium and Reverse Trapezium Block)

   $$a, \beta \geq 0 \quad \text{and} \quad a + \beta \leq 1$$

   (ii) For case (b), (Triangular Stress Block)

   $$0 \leq a \leq 1 \quad \text{and} \quad 0 \leq \beta \leq 1$$

5. Decrease the value of p and repeat steps 3 To 5 until $p = 0$.
6. Increase the value of m and repeat steps 2 to 6 until all possible values of m have been taken.

4 CONCLUSIONS

The following conclusions may be reached from this project,

1. The imperfection approach to column buckling is the only rational method that can be used to analyze and design real columns.
2. Significant reduction in the load carrying capacity of columns can be expected when the column strengths about the major and minor axes are close to each other.
3. Biaxially buckling columns can not be designed safely by using conventional column curves
4. Availability of cheap, fast and efficient computers suggests alternative approaches to column design, with less emphasis on empirical formulae and more emphasis on rational thinking

5 REFERENCES

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